On the decoherence effect of a stochastic gravitational perturbation on scalar matter and the possibility of its interferometric detection

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We present a general master equation for the quantum dynamics of a scalar bosonic particle interacting with an external weak and stochastic gravitational field. The dynamics predicts decoherence in position as well as in momentum. We show how the master equation reproduces the results present in the literature by taking appropriate limits, thus explaining the apparent contradiction in their dynamical description. We apply our model to a matter wave experiment, providing a practical formula for determining of the magnitude of gravitational decoherence. We compare it with the standard experimental sources of decoherence.

Introduction. - The recent exciting first detections of gravitational waves [11, 2], which marked a new era in astrophysics and cosmology, have pushed the scientific community towards the construction of ever more sophisticated ground and space based detectors [3, 4, 7] in order to observe waves in a variety of ranges, possibly down to the cosmic background gravitational radiation. Detecting the latter would open the possibility to gain relevant information about the universe at its very primordial stage, about $10^{-22}$ s after the Big Bang [8], where we expect our description of gravity to fail [9, 10], also because of its unclear relation with quantum matter.

In this scenario, the extreme sensitivity of matter waves [11, 12, 13] to gravity gradients [15] makes matter interferometers a perfect candidate for exploring the gravitational wave background [16, 17] and, at the same time, for possibly answering some fundamental questions regarding the nature of gravity [18, 22], and its coupling to quantum matter.

In this letter we analyse the sensitivity of atom interferometry to a stochastic gravitational background, whose general effect on quantum matter is a path dependent phase shift which ultimately leads to decoherence [23, 24]. There is a rather rich literature on the subject [25, 29], which however seems to yield contradictory predictions for such an effect, in particular regarding the preferred basis of decoherence. Without a clear description of gravitational decoherence, it is not possible to assess if and to which extent matter wave interferometers represent a viable platform to explore the gravitational background. We present a general non relativistic model of gravitational decoherence which clarifies the apparent discrepancies [30]. We show how the results in the literature can be understood as limiting case of our overarching model.

In light of this general result, we characterize the study of the sensitivity of Mach-Zehnder interferometers to a stochastic gravitational background in particular regimes of interest.

The model. - In what follows we report the essential steps of the derivation of the master equation describing gravitational decoherence and its main features. We refer the reader to [30] for a detailed derivation which contains all calculations and references to the mathematical techniques there employed.

To start with, the interaction between matter and gravity is derived from the action of a scalar bosonic matter field $(\phi)$ minimally coupled to the metric $(g_{\mu\nu})$. Under the assumption of small fluctuations of flat spacetime, i.e. $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $|h_{\mu\nu}| \ll 1$, the action is expanded around the Minkowski metric [41] obtaining:

$$S = \int d^4 x \, c^2 (\partial_{\mu} \phi \partial^{\mu} \phi - \frac{m^2 c^2}{\hbar^2} |\phi|^2) - \frac{1}{2} h^{\mu\nu} T_{\mu\nu}^{(0)} + \mathcal{O}(h^2)$$

(1)

from which the equations of motion (EOM) can be easily derived. In this framework the EOM have to be understood as acting on flat spacetime, and the effect of the metric perturbation is expressed via an external force described by the coupling of the gravitational perturbation with the flat matter stress-energy tensor $(T_{\mu\nu}^{(0)})$.

The EOM can be rewritten in terms of the positive and negative energy components of the bosonic field, and the non relativistic limit is taken by means of the Foldy-Wouthuysen transformation [31]. The model is then extended to describe the effects of small spacetime fluctuations on the center of mass dynamics of an extended body of mass $M$. The quantization follows in the canonical way.

As for the gravitational background, we specialize to the case of a gaussian stochastic perturbation. After averaging over the gravitational noise, a master equation for the extended particle is derived (See Eq.(30) of [30]). Here we focus on the case where the stochastic perturbation is a homogeneous, isotropic and white in time noise, with zero mean, and variance $\mathbb{E}[h_{\mu\nu}(x,t)h_{\alpha\beta}(y,s)] = \frac{L_\alpha^2}{c} u_{\mu\nu}(x-y) \delta(t-s)$, where $\alpha$ and $L$ are respectively the strength of the fluctuations and the correlation length. Under these assumptions, the master equation describing the dynamics of a non relativistic matter field in presence of weak spacetime fluctuation reads:
where $\mathbf{X}$ and $\hat{P}$ are the particle’s center of mass position and momentum operators, and $\hat{u}^{\mu\nu}(\mathbf{q})$ and $m(\mathbf{q})$ are respectively the Fourier transform of the noise correlation function and of the particle mass density. The decoherence mechanism described by Eq. (2) is rather complex, contrary to those in the literature [23,29], where only momentum or position decoherence occurs. For this reason, in the next section we study the specific regimes in which only position or momentum decoherence dominate, and show how to recover the existing literature, thus reconciling the apparently contradictory results.

**Recovering position and momentum decoherence.** - The model in Eq. (2) describes decoherence in position when the $h^{00}$ component of the metric fluctuations is at least of the same order of magnitude of the others, i.e.

$$h^{00} \gtrsim h^{0i}, h^{ij}$$

(3)

In this regime we are allowed to neglect the terms containing $h^{0i}, h^{ij}$ [12], thus Eq. (2) simplifies as:

$$\partial_t \hat{\rho} = -i \frac{\hat{P}^2}{\hbar} \hat{\rho}(t) - \frac{\alpha^2 L c^3}{(2\pi)^3/2\hbar^6} \int d^3q \hat{u}^{00}(\mathbf{q}) m^2(\mathbf{q}) \left[ e^{iq \cdot \mathbf{X}/\hbar}, [e^{-iq \cdot \mathbf{X}/\hbar}] \hat{\rho}(t) \right]$$

(4)

Eq. (4) describes indeed a position decoherence process. The above equation replicates the models in [23] and in [29] under the assumption of a pointlike particle:

$$m(\mathbf{r}) = M \delta^3(\mathbf{r})$$

(5)

under the influence of a noise with correlation function:

$$\hat{u}^{00}(\mathbf{q} - \mathbf{q}') = L^3 \hbar^3 \delta(\mathbf{q} - \mathbf{q}') e^{-\hbar^2 \mathbf{q}'^2 L^2/2}$$

(6)

Choosing instead the following form for the correlation function:

$$v^{00}(\mathbf{x} - \mathbf{x}') = L^3 \delta^3(\mathbf{x} - \mathbf{x}')$$

(7)

one can re-obtain the model in [27] with a minor mismatch in the rate functions. Such a mismatch can be accounted to a different treatment of the gravitational perturbation in the two models; in [27] the perturbations are described by a quantum noise, thus allowing for complex correlation functions, while in our case the gravitational noise is classical.

On the other hand, the master equation in Eq. (2) describes decoherence in momentum when the correlation length of the noise is much bigger than the particle’s spatial coherence. In this regime there is a low-momentum transfer from the noise to the matter field, and we are allowed to make the following approximation $e^{i q \cdot \mathbf{X}/\hbar} \sim \hat{1}$ to simplify Eq. (2) as follows:

$$\partial_t \hat{\rho} = -i \frac{\hat{P}^2}{\hbar} \hat{\rho}(t)$$

$$- \frac{\alpha^2 L c^3}{(2\pi)^3/2\hbar^6} \int d^3q \hat{u}^{00}(\mathbf{q}) m^2(\mathbf{q}) \left[ \hat{P}^2, [\hat{P}^2] \hat{\rho}(t) \right]$$

$$- \frac{\alpha^2 L c^3}{(2\pi)^3/2\hbar^6} \int d^3q \hat{u}^{00}(\mathbf{q}) m^2(\mathbf{q}) \left[ \hat{P}^2, [\hat{P}^2] \hat{\rho}(t) \right]$$

(8)

Eq. (8) describes indeed a momentum decoherence process. This equation replicates the model in [28], for a gaussian mass density distribution:

$$m(\mathbf{r}) = \frac{m}{(\sqrt{2\pi} R)^3} e^{-r^2/(2R^2)}$$

(9)

with: $h^{ij} \gg h^{0i}, h^{00}$ and correlation function

$$\hat{u}^{ij}(\mathbf{q} - \mathbf{q}') = \delta^{ij} L^3 \hbar^3 \delta(\mathbf{q} - \mathbf{q}') e^{-\hbar^2 \mathbf{q}'^2 L^2/2}$$

(10)

It also recovers the result in [29] with a minor difference in the rate function, that can again be accounted to the quantum treatment of the gravitational noise in [29].

Our general result shows that the effect of space-time fluctuations on a non relativistic quantum matter can result both in position and/or momentum decoherence depending on the properties of the noise relative to the state of the particle. It also sets the regimes of validity of the models in the existing literature, thus solving the preferred basis puzzle.

In the next section we investigate the sensitivity of a Mach-Zehnder matter wave interferometer to spacetime
stochastic fluctuations. We apply the result to the HYPER experiment 32, a space based, Cesium atom interferometer aiming at testing the weak equivalence principle and measuring the Lense-Thirring effect in the Earth’s gravitational field.

Application: Mach-Zehnder interferometry - In a Mach-Zehnder set up, like the one schematically depicted in Fig. 1, the effect of decoherence is a loss of contrast in the interference pattern produced at the detector 33 34. To quantify this loss we use the interferometric visibility

$$\nu\text{, which is defined in terms of the maximum } (P_{\text{max}}) \text{ and minimum } (P_{\text{min}}) \text{ intensity of the interference pattern: }$$

$$\nu = \frac{P_{\text{max}} - P_{\text{min}}}{P_{\text{max}} + P_{\text{min}}}$$

(11)

We therefore implement a model for the evolution of the probability density to then determine the visibility. Since the condition 33 is usually satisfied for small spacetime fluctuations, we consider the simpler Eq. (4), which we further specialise by means of Eq. (6), as the background gravitational noise is expected to be gaussian correlated with good approximation. We restrict the analysis to point like particles, as Mach-Zehnder interferometry is currently bound to nucleons and atoms due to technical limitations 35. We also assume the matter-wave to be collimated and the interaction time with the mirrors to be negligible, thus we can rely on the longitudinal-eikonal approximation and reduce the study to a one dimensional problem along the transverse axis of propagation, i.e. the x-axis in Fig. 1.

We work with the characteristic function 36 37, which is defined in terms of the statistical operator $\rho_t$ as:

$$\chi_t(s, q) = \frac{1}{2\pi \hbar} \int \langle x - \frac{y}{2} | \rho_t | x + \frac{y}{2} \rangle e^{i(qx + (y-s)p)/\hbar} dy dp$$

(12)

and is connected to the probability density, and thus the interference fringes, through the relation:

$$P_t(x) = \int \chi_t(0, q) e^{-iqx/\hbar} dq$$

(13)

In this formalism free evolution is described by $\chi_t(s, q) = \chi_0(s - \frac{m}{\hbar} t, q)$, and the effect of position decoherence by a multiplicative mask 38 $R(t) = \exp \left[ \int_0^t (\Gamma_g(s - \frac{m}{\hbar} \tau)) d\tau \right]$, where $\Gamma_g$ is the decoherence rate function, which in our study is:

$$\Gamma_g(x) = \frac{2m^2 \alpha^2 e^3 L}{\hbar^2} \left( e^{-\frac{x^2}{2\sigma^2}} - 1 \right)$$

(14)

Thus, the evolution of the system from the first beam splitter to the mirrors is described by:

$$\chi_t(s, q) = R(t)\chi_0(s - \frac{q}{m} t, q)$$

(15)

We model the reflection at the mirrors, following the principles of the "image charge", as the sudden transformation 43:

$$\chi_{t_{\text{ref}}} (s, q) \rightarrow \chi_{t_{\text{ref}}} (-4a - s, -q) + \chi_{t_{\text{ref}}} (4a - s, -q) + 2 \cos \left( \frac{aq}{\hbar} \right) \chi_{t_{\text{ref}}} (-s, -q)$$

(16)

where $2a$ is the distance between the two mirrors. Finally, the evolution from the mirrors to the second beam splitter is described again by means of Eq. (15). This results in the following interference pattern at the screen:

$$P_{\text{scel}}(x) = \int dq \int_0^{t_{\text{ref}}} \left[ e^{i\mathcal{H}_g \tau} e^{i\mathcal{H}_g \tau} \chi_0(\tau - 4a - \frac{2aq}{\hbar}, -q) + e^{i\mathcal{H}_g \tau} e^{i\mathcal{H}_g \tau} \chi_0(4a - \frac{2aq}{\hbar}, -q) + 2 \cos \left( \frac{aq}{\hbar} \right) \chi_0 \left( -\frac{2aq}{\hbar}, -q \right) \right]$$

(17)

that can be used to estimate the visibility in Eq. (11) given the explicit form of the state at the first beam splitter $\chi_0(s, q)$. We choose it to be a gaussian wavepacket of spread $\sigma$ in a superposition of momenta $\pm k$ 44:

$$\chi_0(s, q) = \frac{e^{-\frac{a^2 x^2}{\sigma^2}} - \frac{a^2 x^2}{\sigma^2} \cosh \left( \frac{kq x}{\hbar} \right) + \cos \left( \frac{ks}{\hbar} \right)}{e^{-\frac{a^2 x^2}{\sigma^2}} + 1}$$

(18)

with this initial state the time at which the reflection occurs trivially reads $t_{\text{ref}} = a/v$ where $v = k/m$.

The resulting formula for the visibility is very complicated. However, in the longitudinal-eikonal approximation the spread of the wavepacket is much smaller than the arm of the interferometer, i.e. $\sigma \ll a$, and the formula can be simplified. Indeed, the initial state can be
approximated as $\chi_0(s,q) \simeq \sqrt{\pi} \sigma \delta(s)$, in which case we are able to write the visibility as:

$$\nu \simeq e^{\int_0^{a/v} d\tau \Gamma_{g}(2v\tau)}$$  \hspace{1cm} (19)

with

$$\int_0^{a/v} d\tau \Gamma_{g}(2v\tau) = \frac{\alpha^2 c^3 L}{2v \hbar^2} \left( \sqrt{\frac{2\pi L}{L}} \text{erf} \left( \frac{\sqrt{2}a}{L} \right) - 4a \right)$$  \hspace{1cm} (20)

This formula shows a reduction of the visibility proportional to square of the mass of the particle, meaning that a small increase in the latter will give an important gain in the sensitivity to spacetime fluctuations. For sake of completeness we also analyse the interferometer sensitivity in the regime in which the spacial correlation of the stochastic perturbation is much bigger than the size of the superposition, i.e. $a \ll L$. In this regime Eq. (19) simplifies to:

$$\nu \simeq \exp \left( - \frac{4a^3 \alpha^2 c^3 m^3}{3k\hbar^2} \right)$$  \hspace{1cm} (21)

meaning that in this regime a significant increase on the sensitivity can be obtained by increasing the superposition distance $a$, since the exponent in Eq. (21) is proportional to the cube of $a$. In the opposite regime instead, where the size of superposition is much larger than the spatial correlation of the noise, i.e. $a \gg L$, the visibility simplifies to:

$$\nu \simeq \exp \left( - \frac{2a^2 c^3 L m^3}{k_h^2} \right)$$  \hspace{1cm} (22)

and, as expected, an increase of the size of the superposition will not give any significant improvement in the sensitivity.

To make the study more concrete, we analyse the sensitivity to scalar metric fluctuations of the proposed HYPER experiment, as it is a neat example of possible near future application of atomic Mach-Zehnder interferometers in space. There is another study [39] in the literature concerning the effects of a stochastic gravitational perturbation on HYPER, which however deals specifically with the long wavelength tensorial perturbations constituting the so called Binary Confusion Background.

We accordingly set the parameters of our simulated experiment as reported in Table I.

<table>
<thead>
<tr>
<th>m [kg]</th>
<th>k [Nm/s]</th>
<th>a [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.5 \times 10^{-25}$</td>
<td>$8.8 \times 10^{-28}$</td>
<td>$2.5 \times 10^{-3}$</td>
</tr>
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</table>

We analyse the sensitivity of the experiment for different values of the coherence length $L$ and the strength $\alpha$ of the fluctuations, see Fig. (2).

Our analysis shows a maximum of the sensitivity for of fluctuations with correlation length comparable with the size of the interferometer: $L \approx 10^{-3} \text{m}$, with a strength as small as $10^{-21}$.

The study so far was carried out assuming no other source of decoherence except gravitational fluctuations, while in real experiments different sources of decoherence are always present [40]. We show that the most relevant source of decoherence, i.e. thermal gas collisions, gives a negligible effect. We will not consider other sources of decoherence because they strongly depend on the specific setup. The decoherence function $\Gamma_{\text{coll}}(x-x)$ describing gas collision is not easy to use for a direct estimation of the strength of the effect [40], however in an interferometric experiment usually the superposition distance is much bigger than the typical thermal De Broglie wavelength of the gas allowing one to rely on the simplified expression:

$$\Gamma_{\text{coll}} = \frac{4\pi \Gamma(9/10)}{5 \sin(\pi/5)} \left( \frac{9\pi \beta_c \beta_g I_g I}{64\hbar c_0 (I + I_g)} \right) \frac{p_g^{3/5}}{K_g T_g}$$  \hspace{1cm} (23)

where $T_g, p_g, m_g$ are the temperature, the pressure and the mass of the gas $I, I_g$ are the ionization energies, $\beta_c$ and $\beta_g$ the static polarizabilities of the matter-wave and gas particle and $v_g = \sqrt{2K_g T_g/m_g}$ is the thermal velocity of the gas particle.

Upon plugging in the values of the parameter relative to the experiment, which are summarized in Table I II III we get:

$$\Gamma_{\text{coll}} \approx 7.6 \times 10^{-30} \text{s}^{-1}$$  \hspace{1cm} (24)
which shows that the decoherence induced by thermal gas collisions is indeed negligible with respect to gravitational decoherence on the sensitivity curve of Fig. (2).

Table II: Gas (Hydrogen) parameters.

<table>
<thead>
<tr>
<th>$I_g$ [eV]</th>
<th>$\beta_g$ [$m^3$]</th>
<th>$T_g$ [K]</th>
<th>$P_g$ [Pa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.6</td>
<td>7.42 x 10^{-41}</td>
<td>20</td>
<td>10^{-11}</td>
</tr>
</tbody>
</table>

Table III: Cesium parameters.

<table>
<thead>
<tr>
<th>$I_c$ [eV]</th>
<th>$\beta_c$ [$m^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.89</td>
<td>59.42 x 10^{-30}</td>
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</table>

Summary. - We have presented a general model describing the dynamics of non relativistic quantum matter subject to weak spacetime fluctuations. We have shown that the effect of such fluctuations can result in both position and/or momentum decoherence depending on the specific form of the fluctuation and the state of the quantum system, thus solving the preferred basis puzzle. We have then studied the effect of gravitational decoherence on a Mach-Zehnder interferometer, providing a practical formula to estimate the sensitivity of such a class of experiments to stochastic scalar fluctuations of the metric. We have analysed the most relevant competing decoherence effect, namely thermal gas collisional decoherence, and shown that to be negligible with respect to the gravitational decoherence. Although based on a simplified model, this study shows that matter-wave interferometry is a promising avenue for testing interface of quantum mechanics and gravity and for the detection of the gravitational background.

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[41] Note that by doing so, we are choosing a reference frame where macroscopic rulers and clocks are not appreciably bent by the gravitational field, as it is reasonable to happen in a real experiment.

[42] In the non relativistic limit $c |P| \frac{L^2}{2M} \ll Mc^2$, therefore $c^0 H_{ij} P_i P_j \frac{L^2}{2M} \ll \hbar_{00} Mc^2$.

[43] For the sake of clarity this corresponds to $\psi_{\text{ref}}(x) \rightarrow \psi_{\text{ref}}(2a - x) + \psi_{\text{ref}}(-2a - x)$; $p \rightarrow -p$ in terms of the wavefunction.

[44] For the sake of clarity this corresponds to $\psi_0(x) = \sqrt{\frac{1}{2\sqrt{\pi} \sigma}} e^{-\frac{x^2}{2\sigma^2}} \cos \left( \frac{kx}{\hbar} \right)$ in terms of the wavefunction.