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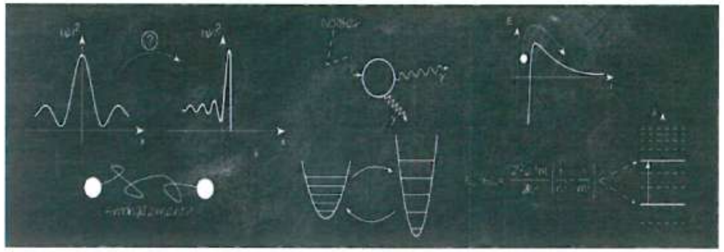

FQXi
FOUNDATIONAL QUESTIONS INSTITUTE

Trieste Junior Quantum Days

A glance in research: where we stand and future challenges

Attendance list - May 18, 2018

#	Name and Surname	Affiliation	Signature
1	Vincenzo Alba	Sissa	[illegible]
2	Linda Anticoli	University of Udine	
3	Vigliano Alessandro Armando	University of Trieste	[illegible]
4	Lorenzo Asprea	University of Trieste	[illegible]
5	Angelo Bassi	University of Trieste	[illegible]
6	Gabriele Bellomia	University of Milano-Bicocca	[illegible]
7	Fabio Benatti	University of Trieste	Fabio Benatti
8	Chendjou Beukam Gervais Nazaire	ICTP-SISSA	[illegible]

9	Francesco Bianco	University of Trieste	
10	Andrea Blason	University of Trieste, SISSA	
11	Riccardo Bonetto	University of Trieste	
12	Fabrizio Caragiulo	University of Trieste - SISSA	
13	Johanna P. Carbone	University of Trieste	
14	Matteo Carlesso	University of Trieste	
15	Alessandro Carotenuto	SISSA	
16	Daniele Ceravolo	University of Trieste	
17	Salvatore Chiavazzo	University of Trieste	
18	Andrea Colcelli	SISSA	
19	Alessandra Colla	University of Padova	
20	Ludovica Crosato	University of Trieste	
21	Raffaele d'Amelio	University of Trieste	
22	Gianni Del Bimbo	LMU	
23	Gianfausto dell'Antonio	University La Sapienza and SISSA	
24	Nicola De Nitti	Università degli Studi di Bari Aldo Moro	
25	Antonello D'Oronzo	University of Trieste	
26	Andrea Drago	Università "La Sapienza" di Roma	
27	Giulio Favaro	University of Padova	

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42	Adam Magee	SISSA	
43	Stefano Mangini	University of Trieste	
44	Gonzalo Manzano Paule	ICTP	
45	Stefano Marcantoni	University of Trieste	
46	Emanuele Marconato	University of Padova	

47	Alessandro Michelangeli	LMU Munich	
48	Laura Natali	LMU München	[illegible]
49	Giuseppe Orsatti	University of Padua	Giuseppe Orsatti
50	Silvia Pappalardi	SISSA, ICTP	
51	Davide Pastorello	University of Trento	[illegible]
52	Giovanni Perosa	University of Trieste	Giovanni Perosa
53	Pietro Pinotti	University of Trieste	Pietro Pinotti
54	Eugenio Pozzoli	University of Trieste - SISSA	
55	Eduardo Quintana Miranda	University of Trieste	
56	Mehdi Ramezani	University of Trieste	
57	Francesco Raso Stoia	Milano Bicocca	
58	Simone Rijavec	University of Trieste	
59	Alice Roitberg	Milano Bicocca	
60	Cesare Rollo	University of Padua	
61	Angelo Russomanno	Scuola Normale Superiore di Pisa, ICTP	
62	Raffaele Scandone	SISSA	[illegible]
63	Irene Solaini	University of Rome, La Sapienza	
64	Alessandra Spigai	ArTS	
65	Alessandro Triacca	University of Rome - La Sapienza	[illegible]

66	Andrea Trombettoni	CNR Trieste	
67	Elidiano Tronchin	University of Padova	
68	Digant Tyagi	University of Trieste	
69	Daan van de Weem	SISSA	
70	Nicolò Venuti	University of Trieste	
71	Chiara Veronese	University of Trieste	Chiara Veronese
72	Michele Vischi	University of Trieste	Michele Vischi
73	Marcello Zanghieri	University of Bologna	
74	Davide Zuliani	University of Padua	
75	FANTINI VERONICA	SISSA	
76	Vahan Hovhannisyan	ANSL, Armenia	
77	Vitale Vincenzo	Università degli Studi di Trieste	
78	FRANCESCA ZARABARA	Università di Trieste	
79	MICHELE STECCONI	SISSA	
80	KENZA HAMMAM	RABAT. University - ICTP	
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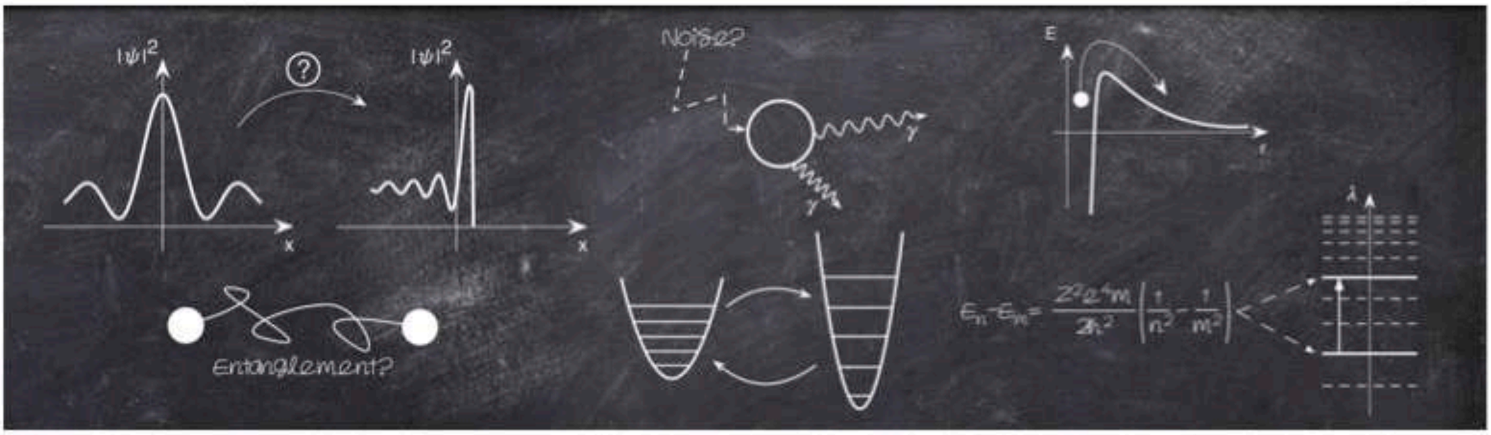
Trieste Junior Quantum Days

A glance in research: where we stand and future challenges



The workshop will gather young researchers working in quantum mechanics and its applications: PhD students and PostDocs from local and nearby institutes will present their research activity. The talks will be pedagogical and easily accessible to master students.

Topics include: quantum information, entanglement, open quantum systems, quantum foundations, many-body physics, quantum thermodynamics, equilibrium & non-equilibrium physics, mathematical methods for quantum mechanics.

The program can be found at the Workshop website: www.tequantum.eu/?q=TriesteQuantumDays/2018

Invited Speakers

Vincenzo Alba (SISSA)
Linda Anticoli (University of Udine)
Matteo Carlesso (University of Trieste)
Andrea Colcelli (SISSA)
Matteo Gallone (SISSA)
Giulio Gasbarri (University of Trieste)
Giacomo Gori (University of Padova)
Stefano Marcantoni (University of Trieste)
Silvia Pappalardi (SISSA, ICTP)
Davide Pastorello (University of Trento)
Angelo Russomanno (Scuola Normale di Pisa and ICTP)
Raffaele Scandone (SISSA)

Scientific Committee

Angelo Bassi (UniTS-INFN)
Fabio Benatti (UniTS-INFN)
Alessandro Michelangeli (LMU Munich)
Andrea Trombettoni (CNR-IOM Trieste)

Local Organizers

Matteo Carlesso (UniTS)
Matteo Gallone (SISSA)

The *Trieste Junior Quantum Days* are sponsored by

TEQ Testing the large-scale limit of quantum mechanics

FQXi FOUNDATIONAL QUESTIONS INSTITUTE

For information: matteo.carlesso@ts.infn.it

Where

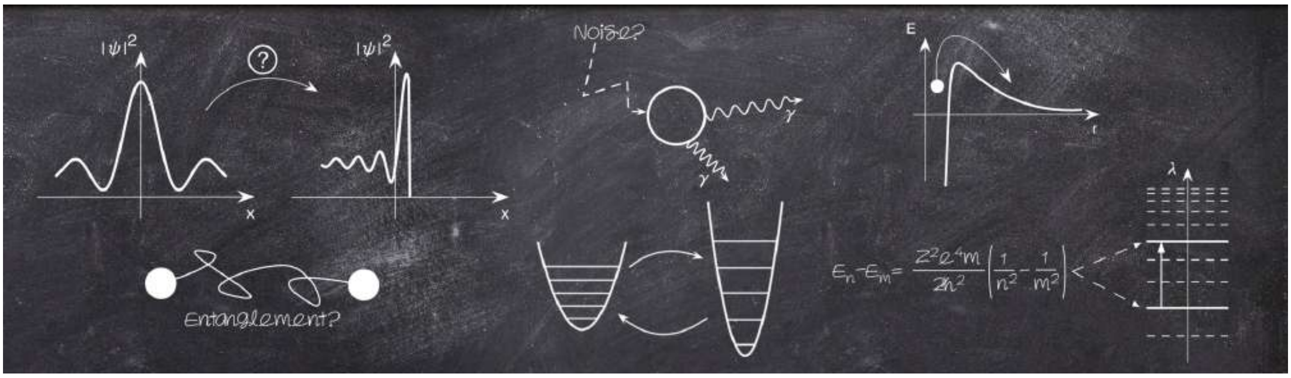
Auditorium - C11 building,
Department of Chemistry,
University of Trieste,
Via Licio Giorgieri 1, Trieste

When

May 11th and **18th**, 2018
14:15 – 19:15pm

Trieste Junior Quantum Days

A glance in research: where we stand and future challenges



Program

May 11th

14:15 Welcome
14:30 **Matteo Carlesso** (University of Trieste)
Can we understand if gravity is quantum?
15:10 **Linda Anticoli** (University of Udine)
Model Checking Recursive Quantum Protocols
15:50 **Andrea Colcelli** (SISSA)
Deviations from Off-Diagonal Long-Range Order and Mesoscopic Condensation in 1D Quantum Systems
16:30 Coffee Break
17:00 **Silvia Pappalardi** (SISSA, ICTP)
Scrambling and entanglement spreading in regular chaotic long range spin chains
17:40 **Giacomo Gori** (University of Padova)
On the performance of a MatterWave based gyroscope
18:20 **Angelo Russomanno** (Scuola Normale di Pisa and ICTP)
Floquet time crystal in the Lipkin-Meshkov-Glick model
19:00 Closing

May 18th

14:15 Welcome
14:30 **Matteo Gallone** (SISSA)
The touchy business of formal computations
15:10 **Giulio Gasbarri** (University of Trieste)
General Galilei covariant Gaussian maps and macroscopicity measure
15:50 **Vincenzo Alba** (SISSA)
Entanglement and thermodynamics after a quantum quench in integrable systems
16:30 Coffee Break
17:00 **Raffaele Scandone** (SISSA)
Non-linear Schroedinger equation with point interactions
17:40 **Davide Pastorello** (University of Trento)
Geometry of Quantum Mechanics in complex projective spaces
18:20 **Stefano Marcantoni** (University of Trieste)
Quantum Model for Impulsive Stimulated Raman Scattering
19:00 Closing
20:00 Dinner

Abstracts

Vincenzo Alba (SISSA)

Entanglement and thermodynamics after a quantum quench in integrable systems

Entanglement and entropy are key concepts standing at the foundations of quantum and statistical mechanics, respectively. In the last decade, the study of quantum quenches revealed that these two concepts are intricately intertwined. Although the unitary time evolution ensuing from a pure initial state maintains the system globally at zero entropy, at long time after the quench local properties are captured by an appropriate statistical ensemble with non zero thermodynamic entropy, which can be interpreted as the entanglement accumulated during the dynamics. Therefore, understanding the post-quench entanglement evolution unveils how thermodynamics emerges in isolated quantum systems. An exact computation of the entanglement dynamics has been provided only for non-interacting systems, and it was believed to be unfeasible for genuinely interacting models. Conversely, here we show that the standard quasiparticle picture of the entanglement evolution, complemented with integrability-based knowledge of the asymptotic state, leads to a complete analytical understanding of the entanglement dynamics in the space-time scaling limit. Our framework requires only knowledge about the steady state, and the velocities of the low-lying excitations around it.

Linda Anticoli (University of Udine)

Model Checking Recursive Quantum Protocols

With the growing interest in the fields of quantum computation and information, the possibility of expressing quantum algorithms, protocols and even quantum dynamics by using an high-level specification language has become crucial. For this reason, we have witnessed the birth of different higher level formalisms allowing to define and simulate automatically formal properties of such protocols, which work by abstracting from low-level physical details. Nevertheless, with the possibility of "programming" quantum protocols comes the need to formally verify them, in order to test that both the specification and the protocols themselves are error-free. To this extent, formal methods such as temporal model checking has been investigated and extended to the quantum domain. We will show our work on Entangle, an integrated framework which provides the possibility to define and automatically verify recursive quantum protocols.

Matteo Carlesso (University of Trieste)

Can we understand if gravity is quantum?

The recent development of interferometric and optomechanical systems gave the opportunity to experimentally approach the long-standing debate whether the gravity has a classical or a quantum intrinsic nature. I will present some of the recent proposals that have been made, highlighting their strong and weak points towards a possible solution.

Andrea Colcelli (SISSA)
Deviations from Off-Diagonal Long-Range Order and Mesoscopic Condensation in One-Dimensional Quantum Systems

A quantum system exhibits off-diagonal long-range order (ODLRO) when the largest eigenvalue $\boldsymbol{\lambda}_0$ of the one-body-density matrix scales as $\boldsymbol{\lambda}_0$~N, where N is the total number of particles. Putting $\boldsymbol{\lambda}_0$~N^C to define the scaling exponent C then C=1 corresponds to ODLRO and C=0 to the single-particle occupation of the density matrix orbitals. When 0< C <1, C can be used to quantify deviations from ODLRO. In this talk I will present the study of the exponent C in a variety of one-dimensional bosonic and anyonic systems.

Matteo Gallone (SISSA)
The touchy business of formal computations

Quantum mechanics requires to deal with unbounded self-adjoint operators on Hilbert spaces which means, in practice, to consider both their action and their domains. Despite that, domain issues are often regarded as "a minor problem". In this talk I will recall the main definitions, I will present with basic examples what can go wrong when one neglects this "minor issue" and I will discuss some more challenging problems including physical Hamiltonians that are central in my research activity: relativistic hydrogen atom, singular potentials and positronium.

Giulio Gasbarri (University of Trieste)
General Galilei covariant Gaussian maps and macroscopicity measure.

Space-time symmetries in open quantum systems have been fully analyzed only in the special, but very important, case of a Markovian, completely positive (CP) and trace preserving (TP) dynamics and the structure of the dynamics fully characterized characterized by Holevo.
This characterization play a major role in the description of several important physical phenomena such as environmental decoherence and relaxation phenomena. Furthermore, it is also relevant for the foundations of quantum mechanics, where an intrinsic non-unitary dynamics is postulated to solve the measurement problem, the black hole information paradox, or to combine principles of general relativity with quantum mechanics.
Although the assumption of Markovianity is often well justified, recent technological advances have led to investigating several phenomena exhibiting memory effects, e.g. ultra fast chemical reactions, side band cooling and light harvesting in photosynthesis.
In this talk we present a complete characterisation for effective non-Markovian Gaussian maps that are Galilei covariant.
We further show how this result can be used to discuss measures of macroscopicity based on classicalization maps, specifically addressing dissipation, Galilean covariance and non-Markovianity.

Giacomo Gori (University of Padova)
On the performance of a MatterWave based gyroscope

We discuss the sensitivity of a guided matter wave interferometer built to measure rotation. We consider the effect of the interaction and temperature on the instrument with different interferometric schemes.

Stefano Marcantoni (University of Trieste)
Quantum Model for Impulsive Stimulated Raman Scattering

Impulsive Stimulated Raman Scattering (ISRS) is a process in which a light pulse is inelastically scattered by a solid sample, exciting vibrations in the latter. This kind of light-matter interaction is usually investigated using time-resolved spectroscopic techniques, in particular pump-probe experiments in which a first intense light pulse, the pump, excites vibrational modes in the crystal and a second less intense light pulse, the probe, is used to test the sample dynamics.
We present a fully-quantum theoretical model that we have recently developed for the description of ISRS in the context of pump-probe experiments. Some preliminary results of this model are validated with measurements performed on quartz.

Silvia Pappalardi (SISSA, ICTP)
Scrambling and entanglement spreading in regular chaotic long range spin chains

We study scrambling, bipartite and multipartite entanglement dynamics in regular and chaotic long range spin chains, with a well dened semi-classical limit. We show that scrambling is a full quantum phenomenon, different from entanglement dynamics. It is characterized by a first semiclassical growth (up to the Ehrenfest time), followed by a fully quantum non-perturbative regime, symmetric around the recurrence time. While entanglement is a state dependent property, we associate scrambling with the growth of the operator's support.

Davide Pastorello (University of Trento)
Geometry of Quantum Mechanics in complex projective spaces

The talk will be focused on the geometric Hamiltonian formulation of quantum mechanics where the projective Hilbert space (as a K hler manifold) plays the role of phase space. Within such a framework quantum observables are represented by phase space functions, quantum states are described by Liouville densities (phase space probability densities), and Schr dinger dynamics is induced by the flow of a Hamiltonian vector field w.r.t. a natural symplectic structure. Then I will discuss how this viewpoint leads to a new approach to quantum control theory based on the Riemannian structure of the projective space.

Angelo Russomanno (Scuola Normale di Pisa and ICTP)

Floquet time crystal in the Lipkin-Meshkov-Glick model

In this talk I will discuss the existence of time-translation symmetry breaking in a kicked infinite-range-interacting clean spin system described by the Lipkin-Meshkov-Glick model. This Floquet time crystal is robust under perturbations of the kicking protocol, its existence being intimately linked to the underlying Z_2 symmetry breaking of the time-independent model. I show that the model being infinite range and having an extensive amount of symmetry-breaking eigenstates is essential for having the time-crystal behavior. In particular, I discuss the properties of the Floquet spectrum, and show the existence of doublets of Floquet states which are, respectively, even and odd superposition of symmetry-broken states and have quasienergies differing of half the driving frequencies, a key essence of Floquet time crystals. Remarkably, the stability of the time-crystal phase can be directly analyzed in the limit of infinite size, discussing the properties of the corresponding classical phase space.

Raffaele Scandone (SISSA)

Non-linear Schroedinger equation with point interactions

A central topic in mathematical physics is the rigorous investigation of many body quantum systems subject to very short range interactions. The dynamics of such systems can be efficiently described by non-linear Schroedinger equations with singular potentials. In this talk, I will discuss a recent result on the well-posedness of the Hartree equation with a point interaction in R^3, in a suitable class of singular Sobolev spaces. I will also discuss various open problems.

Trieste Junior Quantum Days: a big success for the second edition

Quantum mechanics draws big attention, as proved by the success of the workshop coordinated by Angelo Bassi (UniTs-INFN), Fabio Benatti (UniTs-INFN), Alessandro Michelangeli (LMU Munich) and Andrea Trombettoni (CNR-IOM Trieste) and, as local organizers, Matteo Carlesso (UniTs-INFN) e Matteo Gallone (SISSA).

On Friday, May 11, the second edition of the workshop Trieste Junior Quantum Days, was held. The event is conceived as a platform for young students and researchers to discuss on research matters in quantum physics. This year's edition received an even more enthusiastic response compared to the 2017 edition, with a tripled number of participants and a significant presence of Master students, not only in Physics but also in Chemistry. The *junior* nature of the event has to be highlighted: speakers were PhD or young Postdocs of the major academic institutions of the Region and beyond: The universities of Trieste and Udine, the International School for Advanced Studies (SISSA), the International Centre for Theoretical Physics (ICTP) and the University of Padua.

Compared to the previous edition, the first day of the Trieste Junior Quantum Days 2018 attracted attention and was massively attended by participants beyond the regional borders and Triveneto, proving the national and international interest raised by the workshop.
Among the represented institutions there were: University of Milano-Bicocca, University La Sapienza of Rome, University of Bologna, University of Bari *Aldo Moro* and LMU of Munich (Germany).

The second date of the Trieste Junior Quantum Days is Friday, May 18, with a new round of six speakers. For more information, please visit http://tequantum.eu/.

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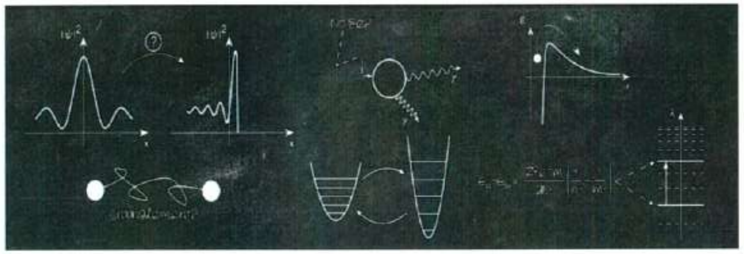

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72	Davide Zuliani	University of Padua	Davide Zuliani
73	Marco Barabella	Univ. di Udine	
74	CLAUDIO MEDICA	TRIESTE	
75	Adu Offei-Danso	SISSA	
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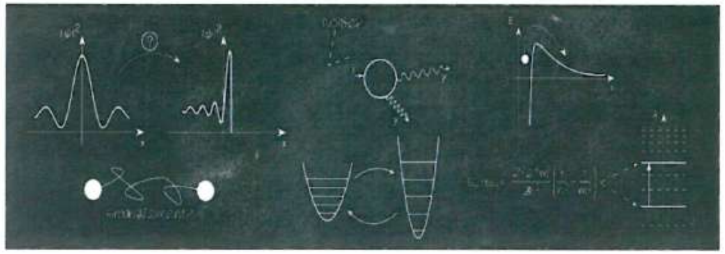

Trieste Junior Quantum Days

A glance in research: where we stand and future challenges

Dinner - May 18, 2018

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59	Alice Roitberg	Milano Bicocca	
60	Cesare Rollo	University of Padua	
61	Angelo Russomanno	Scuola Normale Superiore di Pisa, ICTP	
62	Raffaele Scandone	SISSA	
63	Irene Solaini	University of Rome, La Sapienza	
64	Alessandra Spigai	ArTS	
65	Alessandro Triacca	University of Rome - La Sapienza	

X	66	Andrea Trombettoni	CNR Trieste	Trombettoni Andr.
X	67	Elidiano Tronchin	University of Padova	
X	68	Digant Tyagi	University of Trieste	Digt..
	69	Daan van de Weem	SISSA	
X	70	Nicolò Venuti	University of Trieste	
	71	Chiara Veronese	University of Trieste	Chiara Veronese
	72	Michele Vischi	University of Trieste	
	73	Marcello Zanghieri	University of Bologna	
X	74	Davide Zuliani	University of Padua	
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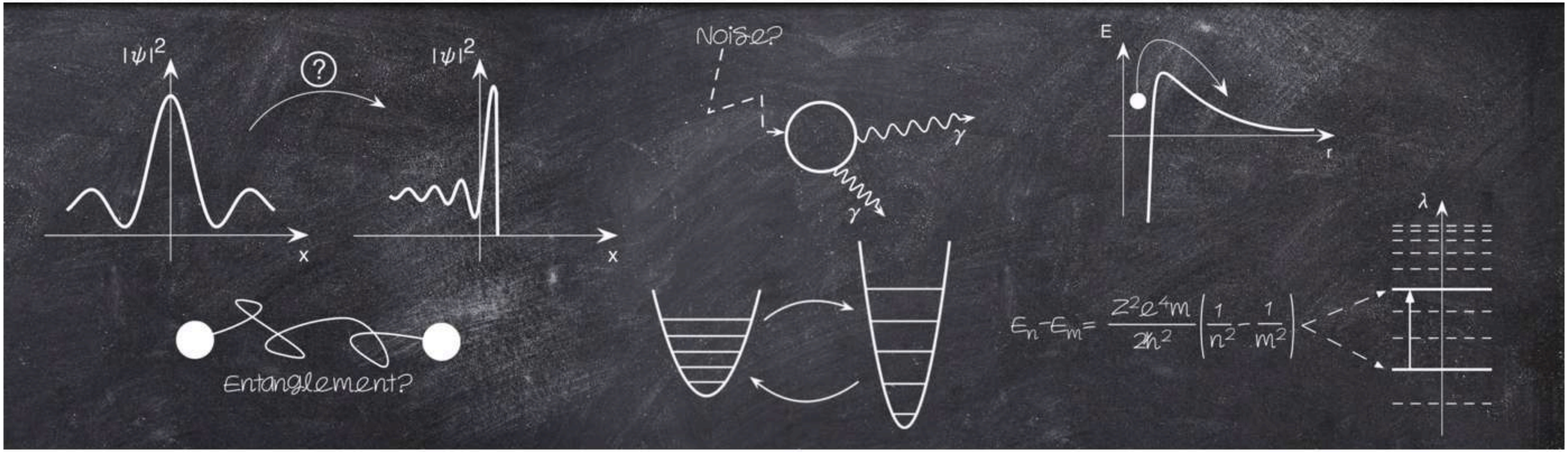
Trieste Junior Quantum Days

A glance in research: where we stand and future challenges



TEQ

HORIZON 2020

FQXi
FOUNDATIONAL QUESTIONS INSTITUTE

Trieste Junior Quantum Days

A glance in research: where we stand and future challenges

14:30	**Matteo Carlesso** (University of Trieste)	*Can we understand if gravity is quantum?*
15:10	**Linda Anticoli** (University of Udine)	*Model Checking Recursive Quantum Protocols*
15:50	**Andrea Colcelli** (SISSA)	*Deviations from Off-Diagonal Long-Range Order and Mesoscopic Condensation in 1D Quantum Systems*
16:30	Coffee Break	
17:00	**Silvia Pappalardi** (SISSA, ICTP)	*Scrambling and entanglement spreading in regular chaotic long range spin chains*
17:40	**Giacomo Gori** (University of Padova)	*On the performance of a MatterWave based gyroscope*
18:20	**Angelo Russomanno** (Scuola Normale di Pisa and ICTP)	*Floquet time crystal in the Lipkin-Meshkov-Glick model*
19:00	Closing	

Trieste Junior Quantum Days

A glance in research: where we stand and future challenges

14:30	**Matteo Gallone** (SISSA)	*The touchy business of formal computations*
15:10	**Vincenzo Alba** (SISSA)	*Entanglement and thermodynamics after a quantum quench in integrable systems*
15:50	**Raffaele Scandone** (SISSA)	*Non-linear Schroedinger equation with point interactions*
16:30	Coffee Break	
17:00	**Davide Pastorello** (University of Trento)	*Geometry of Quantum Mechanics in complex projective spaces*
17:40	**Stefano Marcantoni** (University of Trieste)	*Quantum Model for Impulsive Stimulated Raman Scattering*
19:00	Closing	
20:00	Dinner	

Trieste Junior Quantum Days

A glance in research: where we stand and future challenges

www.tequantum.eu


TEQ

Testing the large-scale limit of Quantum Mechanics

Home News Activities Research Partners Publications Dissemination Contact Members Area

Quantum mechanics provides, to date, the most accurate understanding of the microscopic world of atoms, molecules and photons. Its success is striking and has given rise to vast applications, from nuclear magnetic resonance to the transistor, from the laser to the most accurate GPS.

Besides being undeniably successful, quantum mechanics is equally undeniably weird. It allows putting a microscopic system in the superposition of two different, perfectly distinguishable configurations at the same time. The law that makes such a case possible is the quantum superposition principle, arguably the most fundamental statement in quantum physics.

The validity of quantum superposition principle at the microscopic level has been confirmed by an enormous amount of very accurate experimental data. However, is this valid only when we consider such elementary quantum systems? Everyday experience seems to suggest so: The macroscopic world that is before our very own eyes seems to elude the richness of quantum superposition states. Why don't we see them behaving quantum mechanically?

Everyday experience seems to suggest so: The macroscopic world that is before our very own eyes seems to elude the richness of quantum superposition states. Why don't we see them behaving quantum mechanically?

TEQ will address such a fundamental quest from an innovative standpoint, supported by a € 4.4M grant awarded by the European Commission. A small particle will be levitated within a well-controlled environment, with low temperature and low vibrations. In such an environment an indirect test of the quantum superposition principle can be performed, by analysing carefully the noise that affects the centre of mass motion of the trapped particle. The measured noise will be compared to theoretical predictions from different models - some of which assume a breakdown of quantum linearity.

The ambition of the project is to establish the ultimate bounds to the validity of the quantum framework, if any.

Latest News and Activities

Activity - Friday, May 11 and 18, 2018

Trieste Junior Quantum Days

A glance in research: where we stand and the future challenges

The workshop will gather young researchers working in quantum mechanics: PhD students and PostDocs from local and nearby institutes will present their research activity. The talks will be pedagogical and easily accessible to master students.

Trieste Junior Quantum Days

A glance in research: where we stand and future challenges

www.tequantum.eu/?q=TriesteQuantumDays/2018

TEQ Testing the large-scale limit of quantum mechanics

FQXi FOUNDATIONAL QUESTIONS INSTITUTE

Best Speaker 18th May 2018

Email *

Best Speaker *

- Select -

Submit

Vote for the best speaker!

Leave a feedback

What di you like about the workshop? (The organization, the lecture room, the time given to the speakers...)

What did you not like about the workshop? (The organization, the lecture room, the time given to the speakers...)

What would you like to see for next year's edition? (More/less days, longer/shorter talks...)

Submit

Trieste Junior Quantum Days

A glance in research: where we stand and future challenges

www.tequantum.eu/?q=TriesteQuantumDays/2018

TEQ Testing the large-scale limit of quantum mechanics

FQXi Foundational Questions Institute

Best Speaker 18th May 2018

Email *

Best Speaker *

- Select -

Submit

Leave feedback!

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Submit

Trieste Junior Quantum Days

A glance in research: where we stand and future challenges

BUS 17

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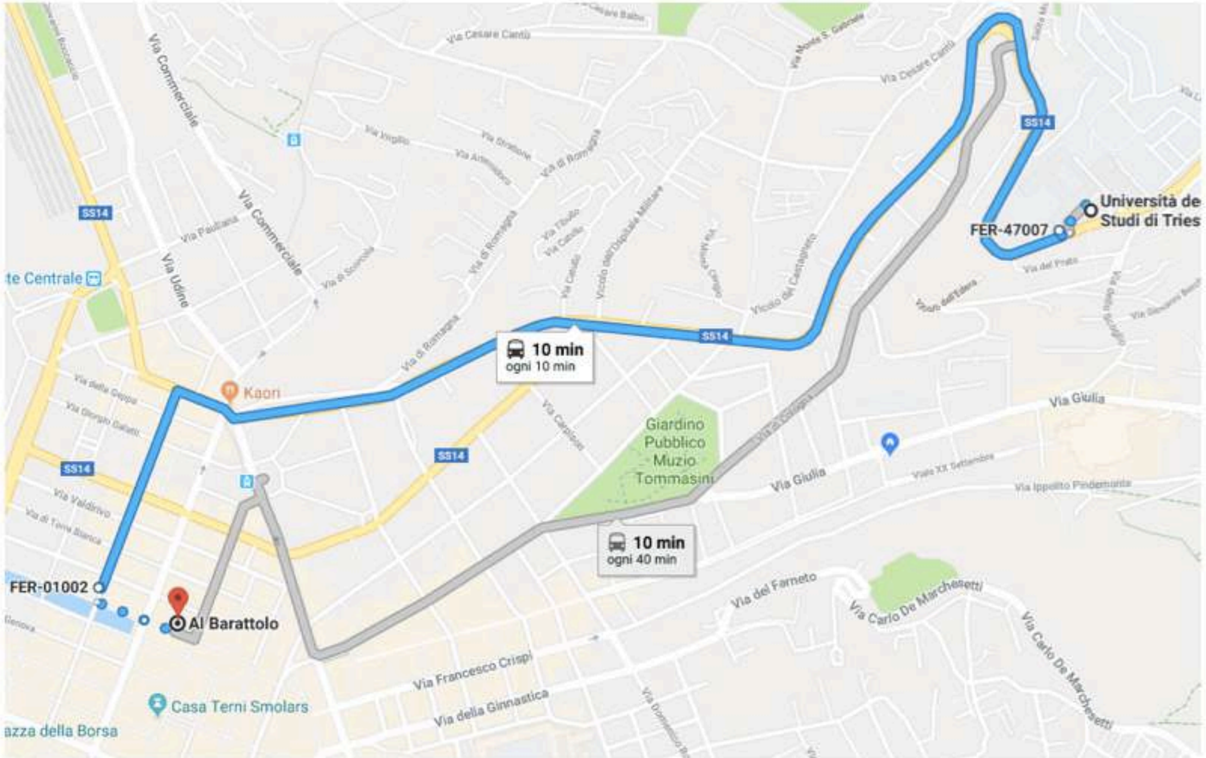

TEQ

HORIZON 2020

FQXi
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Can we understand if gravity is quantum?

Trieste Junior Quantum Days

11th May 2018

Matteo Carlesso (University of Trieste & INFN)

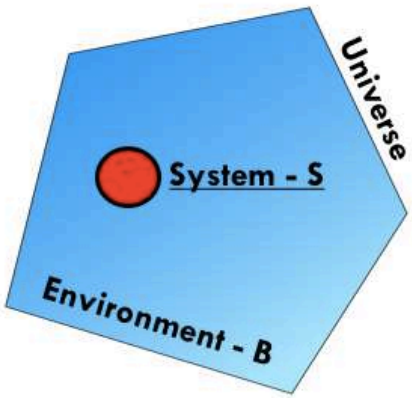
Who am I?

Former student (Bachelor, Master and PhD) and now PostDoc @ University of Trieste

Research Activities

Open Quantum Systems

- Decoherence models
- Collapse models



- MC and A. Bassi.
  Physics Review A, 95, 052119 (2017).
- MC and A. Bassi.
  Physics Letters A, 380, 31–32, pp. 2354 – 2358 (2016).
- S. McMillen, M. Brunelli, MC, A. Bassi,
  H. Ulbricht, M.G. Paris and M. Paternostro.
  Physical Review A, 95, 012132 (2017).
- MC, A. Bassi, P. Falferi, and A. Vinante.
  Physical Review D, 94, 124036 (2016).
- A. Vinante, R. Mezzena, P. Falferi, MC and A. Bassi.
  Physics Review Letters, 119, 110401 (2017).
- MC, M. Paternostro, H. Ulbricht, A. Vinante and A. Bassi.
  ArXiv 1708.04812 (2017)

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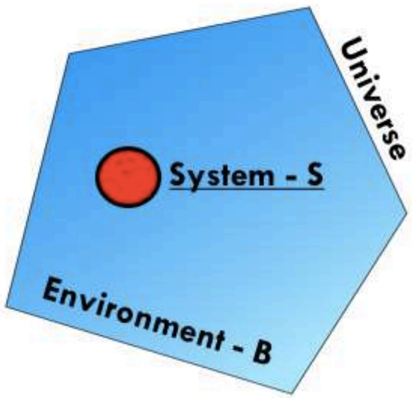


- MC, M. Paternostro, H. Ulbricht and A. Bassi. *ArXiv 1710.08695* (2017)

Gravitational Decoherence
Bassi, Grossardt, Ulbricht,
Class. Quantum Grav. **34**, 193002 (2017)

Review on theoretical and experimental gravity-related works

Several other works, see next slides

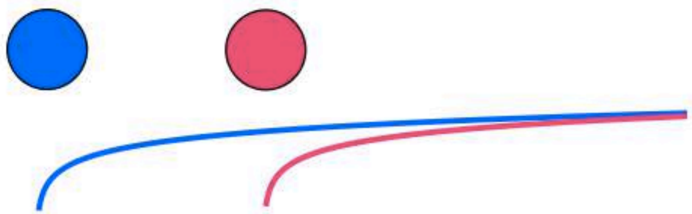
Can we understand if gravity is quantum?

Feynman **The Role of Gravitation in Physics**

Report from the 1957 Chapel Hill Conference

What is the gravitational field generated by a massive quantum superposition?

- Is it the superposition of the two gravitational fields generated by the two terms of the superposition?
- Is it the sum of the two gravitational fields, as predicted by the Schroedinger–Newton equation and perhaps by any theory, which keeps gravity fundamentally classical?



$$i\hbar \frac{\partial \psi(\boldsymbol{r},t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 - Gm^2 \int \mathrm{d}\boldsymbol{r}' \, \frac{|\psi(\boldsymbol{r}',t)|^2}{|\boldsymbol{r}'-\boldsymbol{r}|} \right) \psi(\boldsymbol{r},t)$$

Diosi L 1984 Phys. Lett. A 105

Penrose R 1996 Gen. Relativ. Gravit. 28 581–600

Can we understand if gravity is quantum?

- Is it the superposition of the two gravitational fields generated by the two terms of the superposition?

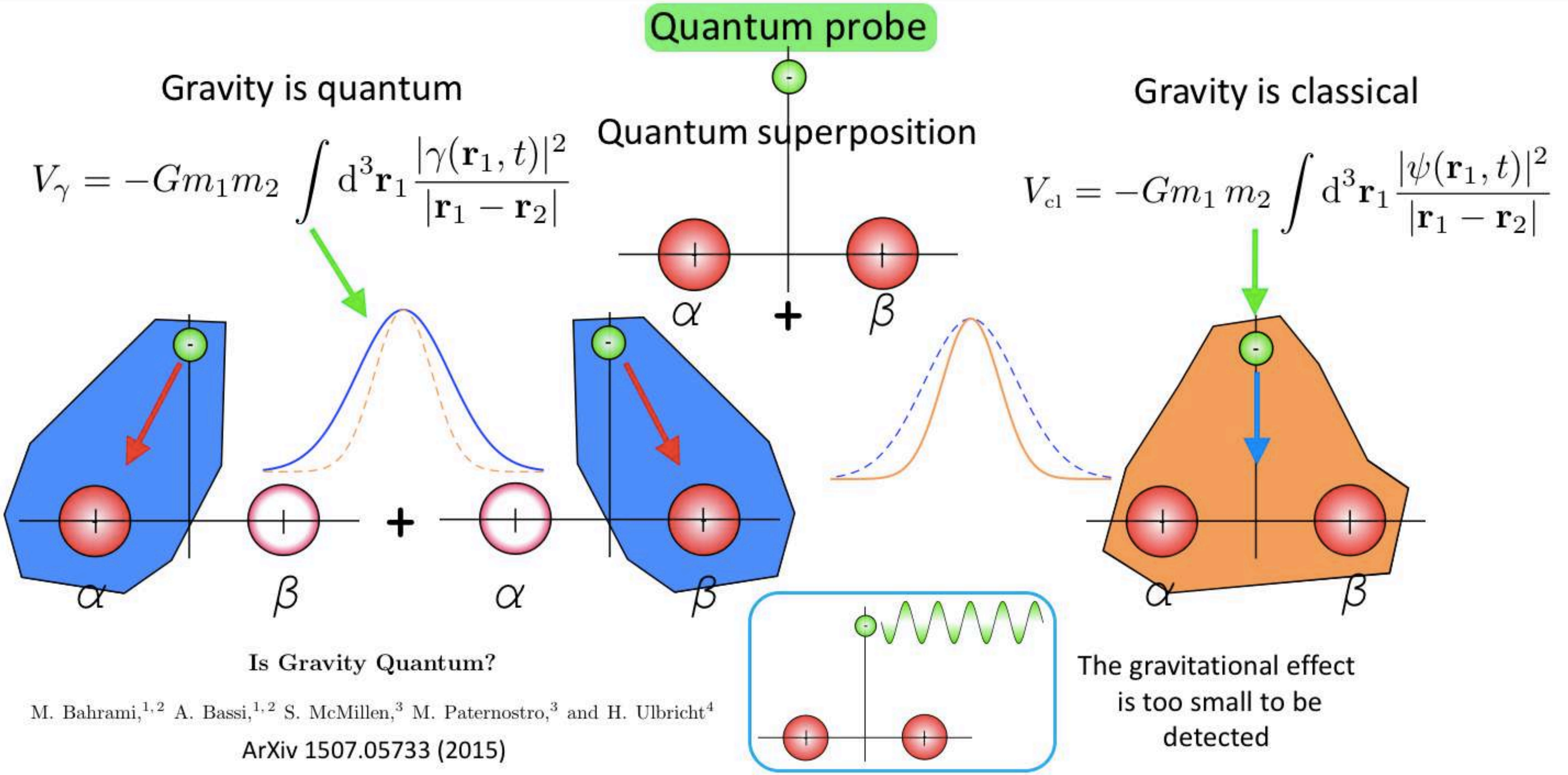
→ Gravitational field is in a superposition;
<u>Quantum scenario</u>

- Is it the sum of the two gravitational fields, as predicted by the Schroedinger–Newton equation and perhaps by any theory, which keeps gravity fundamentally classical?

→ Gravitational field is equally distributed on the superposition;
<u>Classical scenario</u>

→ We propose an experimental scheme to provide evidences in favour or against the quantumness of gravity

Previous experimental proposals



Is Gravity Quantum?

M. Bahrami,[1,2] A. Bassi,[1,2] S. McMillen,[3] M. Paternostro,[3] and H. Ulbricht[4]

ArXiv 1507.05733 (2015)

Previous experimental proposals

The Role of Gravitation in Physics

Report from the 1957 Chapel Hill Conference Feynman

IOP Publishing — Classical and Quantum Gravity
Class. Quantum Grav. **32** (2015) 165022 (24pp) doi:10.1088/0264-9381/32/16/165022

Probing a gravitational cat state

C Anastopoulos[1,3] and B L Hu[2]

PRL **116**, 161303 (2016) PHYSICAL REVIEW LETTERS week ending 22 APRIL 2016

Testing Quantum Gravity Induced Nonlocality via Optomechanical Quantum Oscillators

Alessio Belenchia,[1,*] Dionigi M. T. Benincasa,[1,†] Stefano Liberati,[1,‡] Francesco Marin,[2,3,§] Francesco Marino,[4,‖] and Antonello Ortolan[5,¶]

Is Gravity Quantum? 2015

M. Bahrami,[1,2] A. Bassi,[1,2] S. McMillen,[3] M. Paternostro,[3] and H. Ulbricht[4]

ArXiv 1507.05733 (2015)

PRL **119**, 120402 (2017) PHYSICAL REVIEW LETTERS week ending 22 SEPTEMBER 2017

Revealing Nonclassicality of Inaccessible Objects

Tanjung Krisnanda,[1] Margherita Zuppardo,[1,2] Mauro Paternostro,[3] and Tomasz Paterek[1,4,5]

Witness gravity's quantum side in the lab 2017

Physicists should rethink interference experiments to reveal whether or not general relativity follows classical theory, argue **Chiara Marletto** and **Vlatko Vedral**.

PRL **119**, 240401 (2017) PHYSICAL REVIEW LETTERS week ending 15 DECEMBER 2017

Spin Entanglement Witness for Quantum Gravity

Sougato Bose,[1] Anupam Mazumdar,[2] Gavin W. Morley,[3] Hendrik Ulbricht,[4] Marko Toroš,[4] Mauro Paternostro,[5] Andrew A. Geraci,[6] Peter F. Barker,[1] M. S. Kim,[7] and Gerard Milburn[7,8]

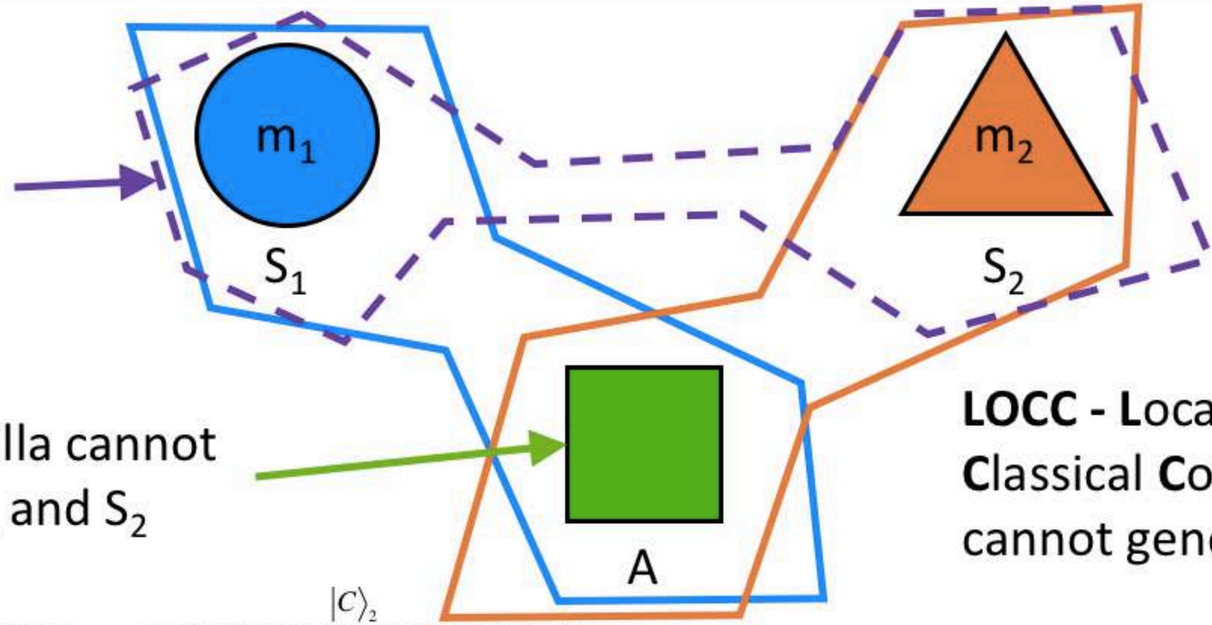
PRL **119**, 240402 (2017) PHYSICAL REVIEW LETTERS week ending 15 DECEMBER 2017

Gravitationally Induced Entanglement between Two Massive Particles is Sufficient Evidence of Quantum Effects in Gravity

C. Marletto[1] and V. Vedral[1,2]

Gravity entangles masses

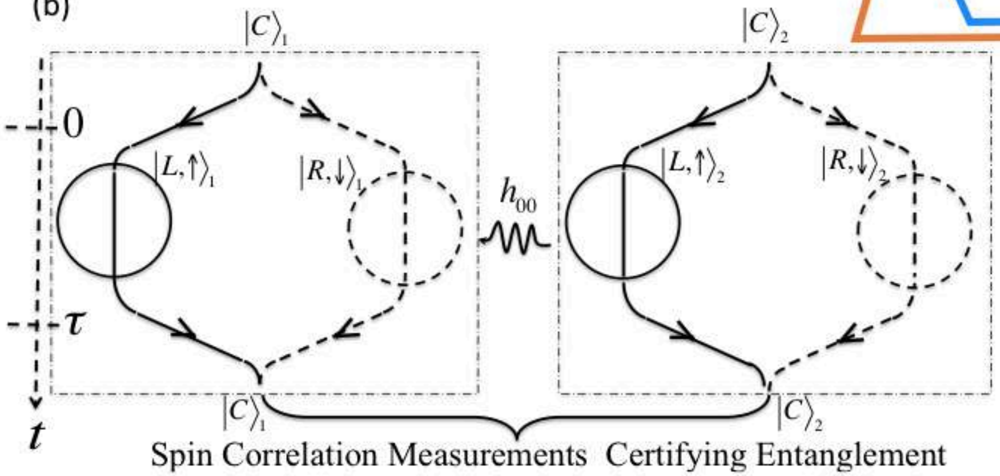
If entanglement is measured, gravity is quantum



A classical ancilla cannot entangle S_1 and S_2

LOCC - Local **O**peration and **C**lassical **C**ommunication cannot generate <u>Entanglement</u>

(b)



PRL **119**, 240401 (2017) PHYSICAL REVIEW LETTERS week ending 15 DECEMBER 2017

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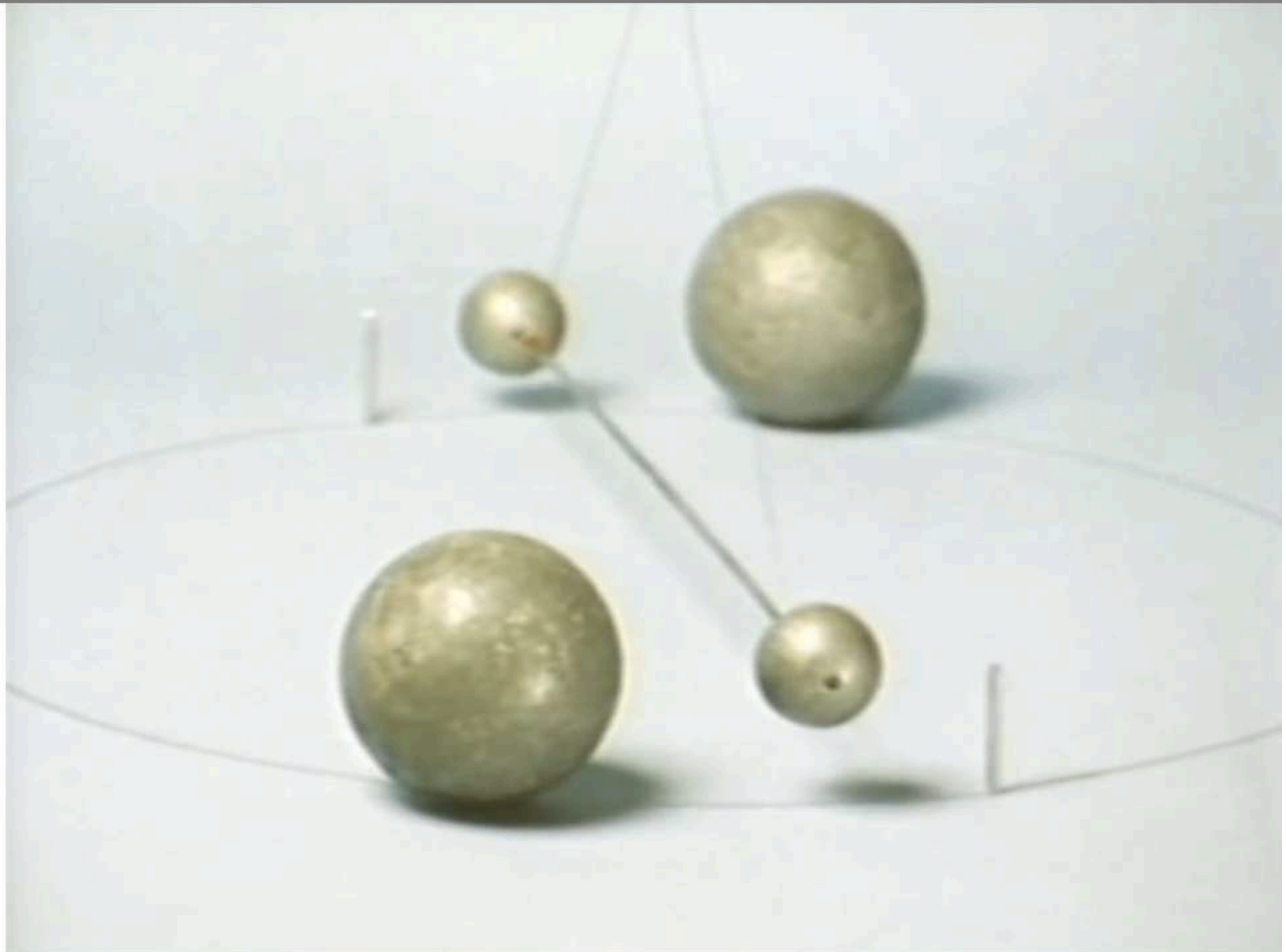
PRL **119**, 240402 (2017) PHYSICAL REVIEW LETTERS week ending 15 DECEMBER 2017

Gravitationally Induced Entanglement between Two Massive Particles is Sufficient Evidence of Quantum Effects in Gravity

C. Marletto[1] and V. Vedral[1,2]

1798 – Cavendish probes Newton's law



2018 – Cavendish probes Feynman???

When Cavendish meets Feynman: A quantum torsion balance for testing the quantumness of gravity

Matteo Carlesso,[1,2,*] Mauro Paternostro,[3,4] Hendrik Ulbricht,[5] and Angelo Bassi[1,2]

ArXiv 1710.08695 (2017)

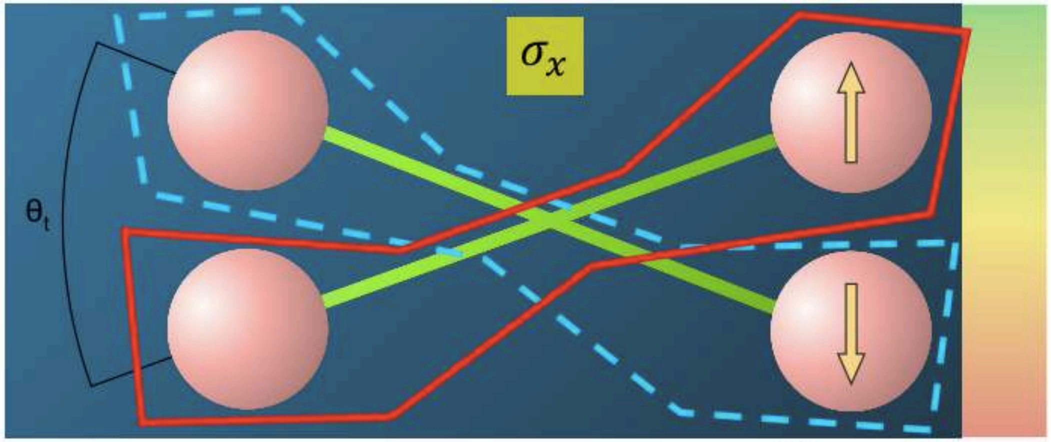
1) Cooling at low Temperature and Pressure

3) Angular superposition

4) Decoupling spin-angular dof

Spin 1 Nitrogen Vacancy

Nanorod



2) MW π/2 pulse

Spin superposition

Magnetic gradient

5) Detection

<u>Quantum scenario</u>

$$V_\gamma = -Gm_1m_2 \int \mathrm{d}^3\mathbf{r}_1 \frac{|\gamma(\mathbf{r}_1,t)|^2}{|\mathbf{r}_1-\mathbf{r}_2|} \longrightarrow$$

No effect, the superposition holds

2018 – Cavendish probes Feynman???

When Cavendish meets Feynman: A quantum torsion balance for testing the quantumness of gravity

Matteo Carlesso,[1,2,*] Mauro Paternostro,[3,4] Hendrik Ulbricht,[5] and Angelo Bassi[1,2]

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1) Cooling at low Temperature and Pressure

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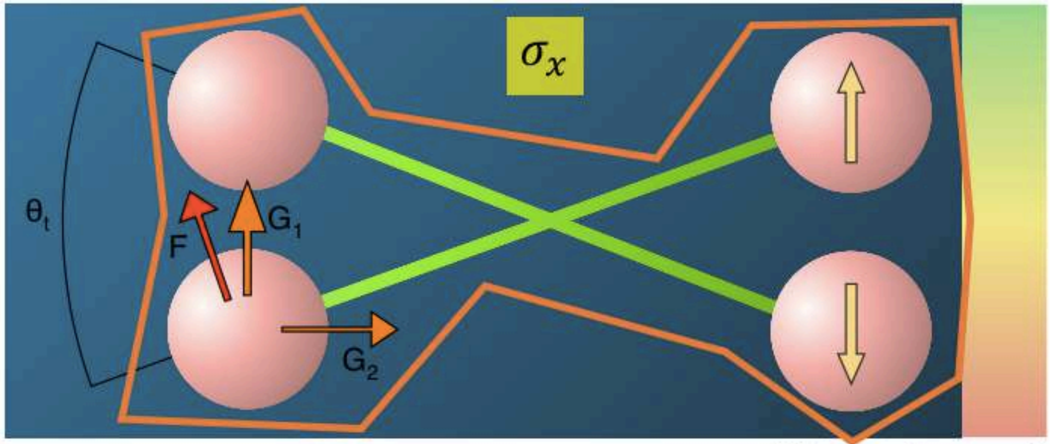
Nanorod



2) MW π/2 pulse
Spin superposition

Magnetic gradient

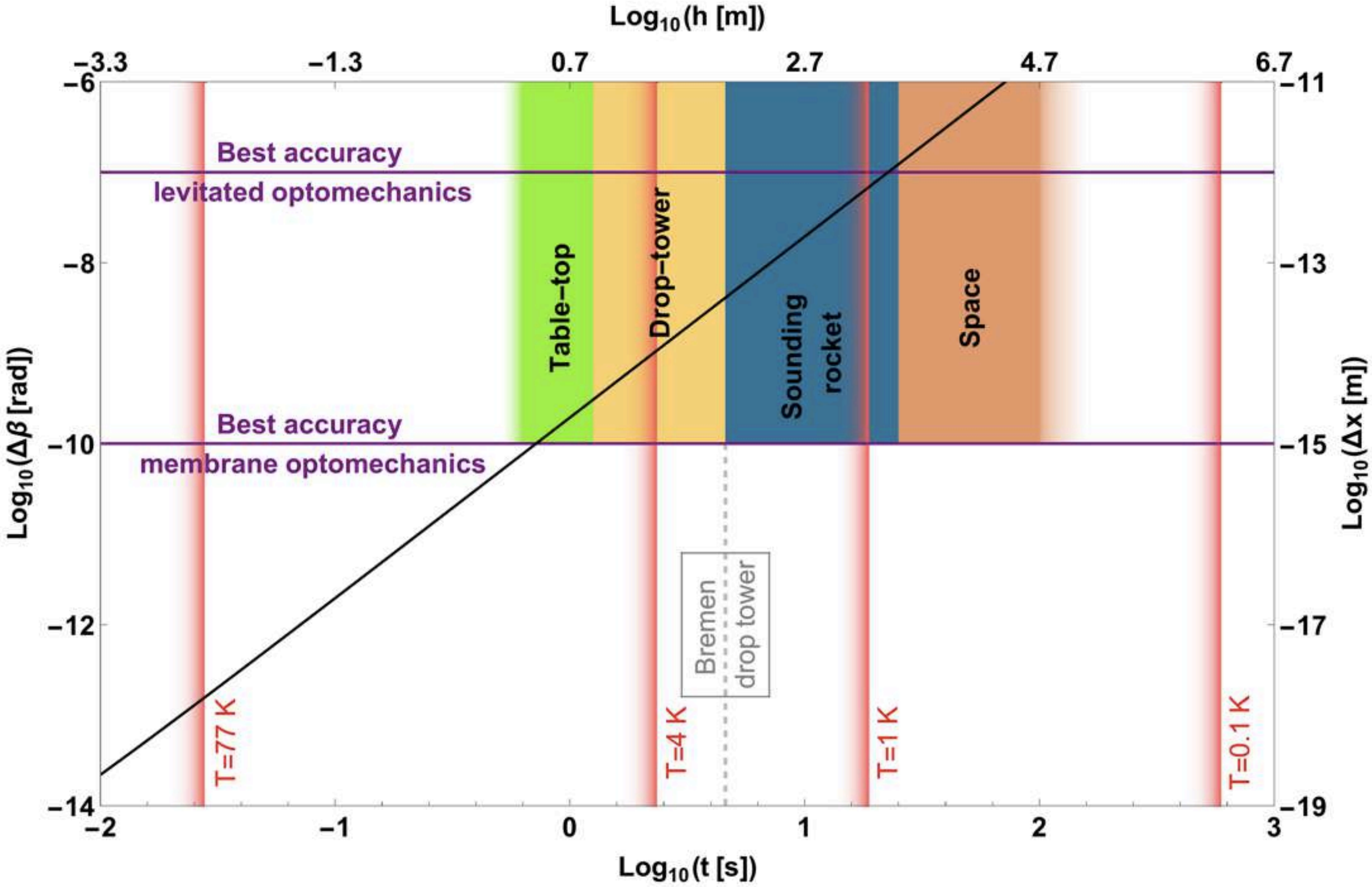
5) Detection

<u>Classical scenario</u>

$$V_{\rm cl} = -G m_1 m_2 \int \mathrm{d}^3\mathbf{r}_1 \frac{|\psi(\mathbf{r}_1,t)|^2}{|\mathbf{r}_1-\mathbf{r}_2|} \longrightarrow$$

Attraction between the two parts of the superposition

Decoherence vs Gravitational effect



Diamond
r=7.92 nm
L=20 um

Conclusions

A test of quantumness of gravity within reach of state-of-the-art technology

- Single self-probing system
    - No limitations in distances
    - Gravitational interaction can be directly observed
- Superposition of torsional degrees of freedom
    - Enhanced measurement precision

Bassi A *et al.,* Class. Quantum Grav. 34, 193002 (2017)

TEQ

HORIZON 2020

UNIVERSITÀ DEGLI STUDI DI TRIESTE

Bassi Group University of Trieste QUANTUM MECHANICS

INFN Istituto Nazionale di Fisica Nucleare

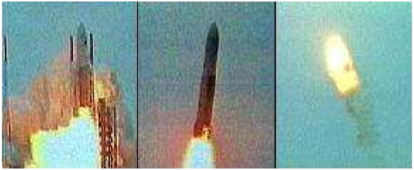
cost EUROPEAN COOPERATION IN SCIENCE AND TECHNOLOGY

QTSPACE

Model Checking Recursive Quantum Protocols

Linda Anticoli

Dept. of Mathematics, Computer Science and Physics - University of Udine, Italy.
School of Computing Science - Newcastle University, UK.



June 1996
Ariane 5 launcher failure

"Loss of information due to specification and design errors in the software of the inertial reference system."

Motivations

Quantum information is more fragile than classical one

$$\Downarrow$$

flaws in the design of quantum protocols and noise in their physical implementation

Motivations

State of the art:

- Formal, higher–level specification of quantum algorithms
  Quantum Programming Languages

  - J. W. Sanders and P. Zuliani. "Quantum Programming" (2000)
  - A. van Tonder. "A lambda calculus for quantum computation" (2003)
  - A.S. Green, , et Al. "Quipper: A Scalable Quantum Programming Language" (2013).

- Formal verification of quantum algorithms
  Quantum Model Checkers

  - P. Mateus, et Al. "Towards model-checking quantum security protocols" (2007)
  - Y. Feng, et Al. "QPMC: A Model Checker for Quantum Programs and Protocols" (2015)

Motivations

Desiderata:

High-level formalisms allowing to define and automatically verify formal properties of algorithms abstracting away from low–level physical details:

- L. Anticoli, et Al. "Towards quantum verification: From Quipper circuits to QPMC" (2016)
- L. Anticoli,et Al. "Entangλe: A Translation Framework from Quipper Programs to Quantum Markov Chains" (2017).

Preliminaries and Notation

Question 1

What is a quantum algorithm (or quantum protocol)?

$\Downarrow$

Quantum Computation and Information

Question 2

What does model–checking mean?

$\Downarrow$

Formal Methods in Computer Science

Quantum Computation and Information – Remarks

Paradigm of computation concerned with computational tasks, and information processing achieved through quantum mechanical systems.

$\boxed{\text{Efficient solutions for classically hard problems}}$

- Integer Factoring $n = log_2 N$
  - Classical Solution -> $\approx \exp[O(n^{1/3} log^{2/3} n)]$;
  - Shor's Algorithm -> $\approx O(n^3)$;
- Unsorted Database Search
  - Classical Solution -> $O(N)$;
  - Grover's Algorithm -> $O(N^{1/2})$;

Quantum Computation and Information – Remarks

Efficiency

- **Parallelism**: linearity of space and operators;
- **Interference**: the states *interfere* deleting the "wrong" ones, while increasing the probability of the desired outcome.
- **Correlations**: non–local correlations between the outcomes of measurements performed on different qubit strings.

Qubit

Superposition of States

Quantum analogue of a classical bit. State of a 2-level system:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

where $|\alpha|^2 + |\beta|^2 = 1$ and $\alpha, \beta \in \mathbb{C}$

Quantum Register

Quantum analogue of a classical bit string composed by n–qubits:

$$|\psi_{tot}\rangle = |\psi_1\rangle \otimes \cdots \otimes |\psi_n\rangle$$

allowing 2^n superposed basis states.

Quantum Circuit Model

Quantum Gates

Quantum counterpart of classical logic gates.

$$n \text{ qubits} \longrightarrow \text{quantum gates: } 2^n \times 2^n \text{ unitary operators.}$$

Single Qubit Gates

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Controlled Gates

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

tipically used to create correlations.

Quantum Circuits

Quantum algorithms are represented by quantum circuits in which the computation is realised by the following steps.

1. State preparation;
2. Application of unitary operators;
3. Measurement.

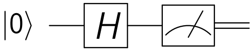


Figure: Quantum Coin Tossing Circuit.

Applications

Quantum Teleportation

Quantum information (qubits) is transmitted from a location to another by means of classical communication and previously shared entangled couples between sender and receiver.

Quantum Cryptography

Use of quantum effects to perform cryptographic tasks.
Measurement disturbs the data –> eavesdropper can be detected!

Refs:

C. H. Bennett, et Al. *"Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels"*

C. H. Bennett, et Al. *"Quantum cryptography: Public key distribution and coin tossing"*

Issues

Problem 1

Quantum circuits are low level descriptions of computation.

Problem 2

Quantum circuits are not Turing complete, no recursion.

Quantum Programming Languages

Solution1

Quantum Programming Languages: abstract the computation from the physical low–level detail to a human readable, and formally defined *high–level* description.

- **Quantum pseudocode**
- **QCL**
- **Q Language**
- **qGCL**
- **Quantum Lambda Calculus**
- **Quipper**
- **LIQUi$|\rangle$**
- ...

Quipper

Functional programming for quantum computation.

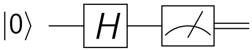
- Based on Haskell;
- The semantics of a Quipper program is given in terms of extended quantum circuits;
- Allows to generate a graphical representation of the implemented circuit, but not of quantum programs;
- Provides three different of simulators.

A Quipper program is a function that inputs some quantum and classical data, performs state changes on it, and then outputs the changed quantum/classical data.

Quipper example

```
qCoinFlip :: Qubit -> Circ Bit
qCoinFlip q = do
  q <- qinit False
  hadamard_at q
  c <- measure q
  return c
```

```
qCoinFlipRec :: Qubit -> recCirc ()
qCoinFlipRec q = do
  q <- qinit False
  hadamard_at q
  c <- measure q
  [...]
  if c==0 then
    return c
  else
    return qCoinFlip(q)
```



Quantum Markov Chains

Solution 2

Data–structures allowing to model recursion in quantum algorithms: Quantum Markov Chains.

Quantum Markov Chain

Tuple (S, Q, AP, L), where:

- S is a countable (finite) set of classical states;
- $Q : S \times S \to S^{\mathcal{I}}(\mathcal{H})$ is the transition matrix where for each $s \in S$,the operator $\sum_{t \in S} Q(s, t)$ is trace-preserving;
- AP is a finite set of atomic propositions;
- $L : S \to 2^{AP}$ is a labelling function.

Example

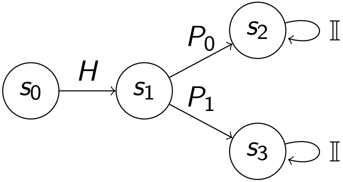


Figure: QMC for Quantum Coin Tossing.

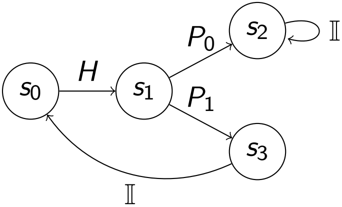


Figure: QMC for Recursive Quantum Coin Tossing.

QMCs are more expressive! So, let's use them.

Equivalent Behaviour

Bisimilarity

A quantum circuit can always be translated in a QMC with the same behaviour, while the converse is not possible.

(Boring proofs in references)

Now what?

- Formal, high–level language to express quantum computations ☑
- Formal definition of recursion in quantum programs ☑
- Formal verification of quantum programs ☐

Quantum Model Checking (1)

Model Checking

Exhaustive exploration of the state space of a system to verify (or falsify) if a temporal property is satisfied.

Step–by–step

- Abstract model of the system;
- Temporal logic to specify the properties.

Quantum Model Checking (2)

Abstract Model

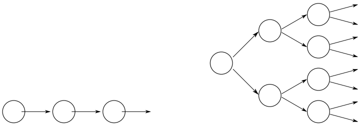
Graph structure representing the computation steps. Classically: Kripke structures, LTS, DTMC.
QMC can be used as a model for quantum computation!

Temporal Logics

Modal logics used to express time–dependent properties.
Example: *"In all the reachable states of the system, property A never holds"*

(a) LTL

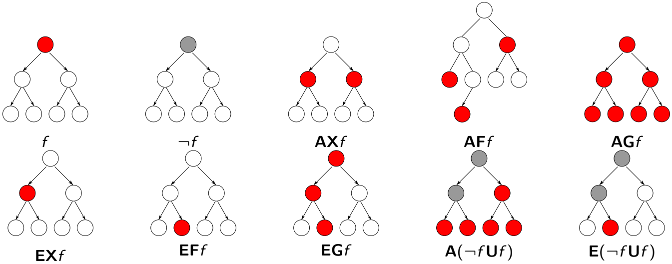
(b) CTL



(a) (b)

Quantum Model Checking (3)

Temporal Operators



Invariant and Eventually

A (i.e., for all computation paths) and **E** (i.e., eventually, for some computation path).

Quantum Model Checking (4)

QCTL

Quantum Computation Tree Logic, it provides also the operators:

- $\mathbb{Q}_{\sim\epsilon}[g]$;
- $Q =?[g]$;
- $qeval((Q =?)[g], \rho)$;
- $qprob((Q =?)[g], \rho) = tr(qeval((Q =?)[g], \rho)))$.

QCTL

Quantum Computation Tree Logic

A QCTL formula is a formula over the following grammar:

$$\Phi ::= a \mid \neg\Phi \mid \Phi \wedge \Phi \mid \mathbb{Q}_{\sim\varepsilon}[\Phi] \qquad \textit{state}\text{ formula}$$

$$\phi ::= X\Phi \mid \Phi U^{\leq k}\Phi \mid \Phi U \Phi \qquad \textit{path}\text{ formula}$$

where $a \in AP$, $\sim \in \{\lesssim, \gtrsim, \eqsim\}$, $\mathcal{E} \in \mathcal{S}^{\mathcal{I}}(\mathcal{H})$, $k \in \mathbb{N}$.

Example

$Q >= 1\ [F(s = 5)]$

What we did: from Circuits to QMCs

Quip-E

We isolated and extended a Quipper fragment that we called *Quip-E* which allows the definition of both standard and *tail recursive* quantum programs.

Entangλe

We defined a mapping from *Quip-E* programs to QMCs. We start by considering a quantum program generated by *Quip-E* and we define a bisimilar QMC.

Formal definition of *Quip-E* program

Definition

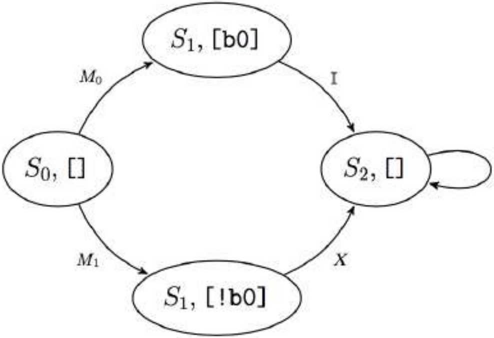
A *Quip-E* program is a circuit in which the result of a measurement is evaluated and could result in a loop.

Body of *Quip-E* program

- **reset**: initializes the qubits to $|0\rangle$;
- **unitary**: unitary operator applied to a list of qubits;
- **measure**: application of measurement operators to a list of qubits resulting in a list of bits;
- **dynamic lift**: A bit is lifted to a boolean through the dynamic lift Quip- per operator;
- **if-then-else**: evaluation of a Boolean expression;
- **exit On**: loop instruction.

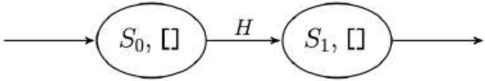
From *Quip-E* to QMC - intuitively

```
resetCirc :: Qubit -> Circ Qubit
resetCirc q = do
  reset_at q
```



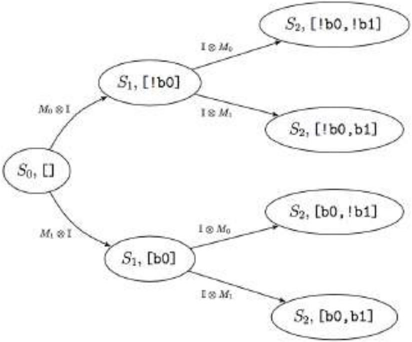
From *Quip-E* to QMC - intuitively

```
unitCirc :: Qubit -> Circ Qubit
unitCirc q = do
  hadamard_at q
```



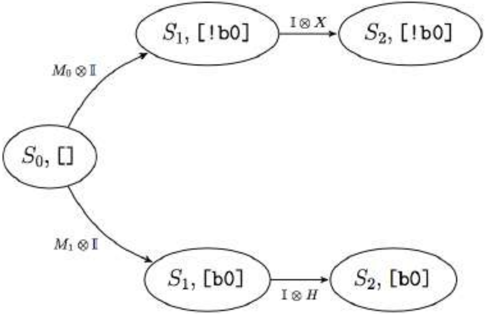
From *Quip-E* to QMC - intuitively

```
measureCirc :: (Qubit, Qubit) -> Circ ()
measureCirc (q1, q2) = do
  c1 <- measure q1
  c2 <- measure q2
```



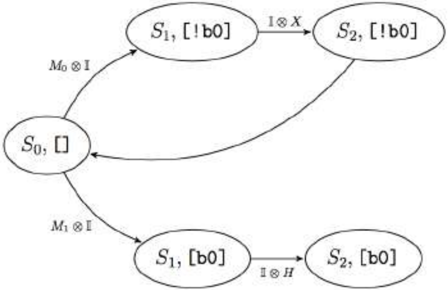
From *Quip-E* to QMC - intuitively

```
iteCirc :: (Qubit, Qubit) -> Circ ()
iteCirc (q1, q2) = do
  c1 <- measure q1
  b0 <- dynamic_lift c1
  if b0
    then do hadamard_at q2
    else do gate_X_at q2
```



From *Quip-E* to QMC - intuitively

```
loopCirc :: (Qubit, Qubit) -> Circ RecAction
loopCirc (q1, q2) = do
  c1 <- measure q1
  b0 <- dynamic_lift c1
  if b0
    then do hadamard_at q2
    else do gate_X_at q2
```



Operational Semantics of Quip–E (1)

$$\frac{}{(\texttt{reset\_at q}_k, L) \xrightarrow{\mathcal{M}_0^k} (\_\_\_\_, L)} \qquad \frac{}{(\texttt{reset\_at q}, L) \xrightarrow{\mathcal{M}_1^k} ((\texttt{X\_at q}_k, L)}$$

$$\frac{}{(\texttt{U\_at [q}_{i_1}\texttt{, ..., q}_{i_j}\texttt{]}, L) \xrightarrow{\mathcal{U}_{i_1,\ldots,i_j}} (\_\_\_\_, L)}$$

$$\frac{}{(\texttt{m} \leftarrow \texttt{measure q}_k, L) \xrightarrow{\mathcal{M}_i^k} (\_\_\_\_, L[L(\texttt{m}) = i]\})} \quad \text{for } i \in \{0, 1\}$$

$$\frac{}{(\texttt{bool <- dynamic\_lift m}, L) \xrightarrow{\mathcal{I}} (\_\_\_\_, L[L(\texttt{bool}) = L(\texttt{m})])}$$

Operational Semantics of Quip–E (2)

$$\frac{L(\texttt{bool}) = i}{(\texttt{if (bool) Body\_C}_1\ \texttt{else Body\_C}_0, L) \xrightarrow{\mathcal{I}} (\texttt{Body\_C}_i, L)} \quad \text{for } i \in \{0,1\}$$

$$\frac{(\texttt{Body\_C}_1, L) \xrightarrow{\mathcal{S}} (\texttt{Body\_C}_1{}', L')}{(\texttt{Body\_C}_1\ \texttt{Body\_C}_2, L) \xrightarrow{\mathcal{S}} (\texttt{Body\_C}_1{}'\ \texttt{Body\_C}_2, L')}$$

$$\frac{(\texttt{Body\_C}_1, L) \xrightarrow{\mathcal{S}} (\_\_\_\_, L')}{(\texttt{Body\_C}_1\ \texttt{Body\_C}_2, L) \xrightarrow{\mathcal{S}} (\texttt{Body\_C}_2, L')}$$

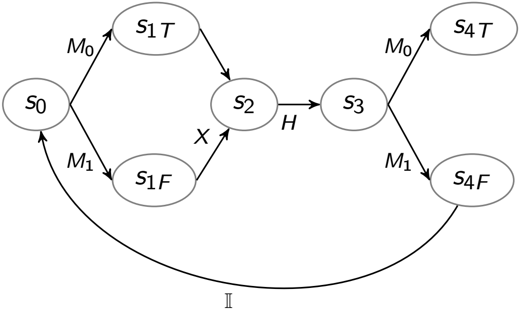
$$\frac{}{(\_\_\_\_, L) \xrightarrow{\mathcal{I}} (\_\_\_\_, L)}$$

Implementation

We implemented `Entang`λ`e` using the `Transformer` module of Quipper. The input quantum program is a *Quip-E* function and the output QMC is a QPMC model.

1. The gates in the quantum circuit are grouped together with their associated qubits, preserving the execution order;
2. we compute the matrix representation of the quantum gates, taking into account also the conditional branches and the initialization operators;
3. the last step is the conversion of the list of transitions into QPMC code.

```
testInit :: (Qubit) -> Circ RecAction
testInit (q) = do
  reset_at q
  hadamard_at q
  ma <- measure q
  bool <- dynamic_lift ma
  exitOn bool
```




```
qmc

const matrix A1_T = M0;
const matrix A1_F = M1;
const matrix A2 = PauliX;
const matrix A3 = Hadamard;
const matrix A4_F = M0;
const matrix A4_T = M1;

module testInit
  s: [0..4] init 0;
  b0: bool init false;

  [] (s = 0) -> <<A1_T>> : (s' = 1) &
        (b0' = true) + <<A1_F>> : (s'
        = 1) & (b0' = false);
  [] (s = 1) & b0 -> (s' = 2);
  [] (s = 1) & !b0 -> <<A2>> : (s' =
        2);
  [] (s = 2) -> <<A3>> : (s' = 3);
  [] (s = 3) -> <<A4_F>> : (s' = 4) &
        (b0' = false) + <<A4_T>> : (s'
        = 4) & (b0' = true);
  [] (s = 4) & !b0 -> (s' = 0);
  [] (s = 4) & b0 -> true;
endmodule
```

Conclusion and Questions

TO–DO

- optimization of `Entang`λ`e` to verify more complex quantum programs;
- optimization from the model checking point of view, involving the automatic verification of more complex properties, i.e., entanglement and other quantum effects;
- translation and verification of more complex, real-world quantum protocols;
- simulation (and translation) of quantum dynamics;
- spatial properties verification.

Deviations from Off-Diagonal Long-Range Order and Mesoscopic Condensation in One-Dimensional Quantum Systems

ma per seguir virtute e canoscenza

SISSA

Andrea Colcelli

11/05/2018

Outline

- Off-Diagonal Long-Range Order (ODLRO)
- ODLRO in 1D: Introduction to the Lieb-Liniger Model
- One-body density matrix for Lieb-Liniger bosons
- One-body density matrix for Lieb-Liniger anyons (including 1D hard-core anyons)
- Conclusions

Outline

- Off-Diagonal Long-Range Order (ODLRO)
  O. Penrose, L. Onsager, *Phys. Rev., 104 (1956) 576*

  L. P. Pitaevskii, S. Stringari, *Bose-Einstein condensation and Superfluidity,* Oxford University Press
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Density Matrix

One-Body Density Matrix (OBDM):

$$H\chi_N(\overrightarrow{r_1},\overrightarrow{r_2},\dots,\overrightarrow{r_N}) = E\chi_N(\overrightarrow{r_1},\overrightarrow{r_2},\dots,\overrightarrow{r_N})$$

$$\rho(\vec{r},\vec{r}\,') \equiv N\int d\overrightarrow{r_2}\dots d\overrightarrow{r_N}\;\chi_N^*(\vec{r},\overrightarrow{r_2},\dots,\overrightarrow{r_N})\;\chi_N(\vec{r}\,',\overrightarrow{r_2},\dots,\overrightarrow{r_N})$$

Diagonal OBDM:

$$\rho(\vec{r}) \equiv \rho(\vec{r},\vec{r})$$

Normalization $\int \rho(\vec{r})\,d\vec{r} = N$

Off-Diagonal Long-Range Order

$$\int \rho(\vec{r},\vec{r}')\,\varphi_j(\vec{r}')\,d\vec{r}' = \lambda_j\,\varphi_j(\vec{r})\,, \qquad \lambda_j \in \mathbb{R}, \qquad \sum_j \lambda_j = N, \qquad \lambda_0 \geq \lambda_1 \geq \lambda_2 \geq \cdots$$

natural orbitals occupation numbers

$$\int \varphi_j(\vec{r})\,\varphi_k(\vec{r})\,d\vec{r} = \delta_{j,k}\,, \quad \sum_j \varphi_j(\vec{r})\,\varphi_j(\vec{r}') = \delta(\vec{r}-\vec{r}')$$

$$\rho(\vec{r},\vec{r}') = \sum_i \lambda_i \varphi_i^*(\vec{r})\varphi_i(\vec{r}')$$

$\lambda_0 \propto N$ $\boxed{\text{ODLRO}}$ $\lambda_0 \propto N^0$ $\boxed{\lambda_0 \propto N^{\mathcal{C}}}$ $0 \leq \mathcal{C} \leq 1$ $\boxed{\text{Deviations from ODLRO}}$

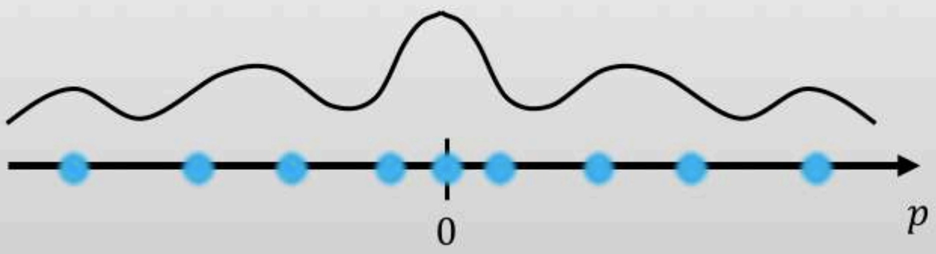
$\boxed{\text{BEC}}$ $\boxed{\text{Fermions}}$

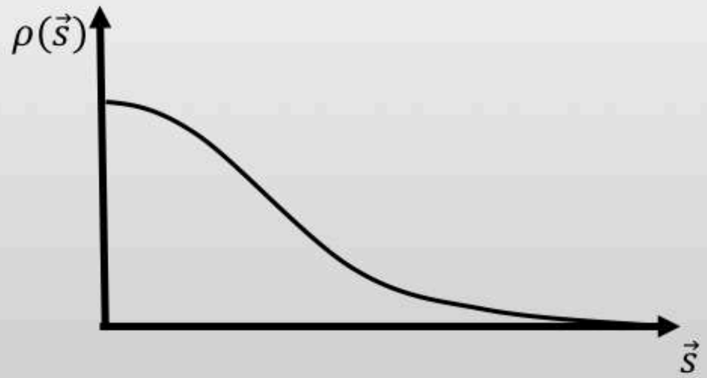
Momentum distribution

$$n(\vec{p}) = \frac{1}{(2\pi\hbar)^3}\int d\vec{r}'\int \rho(\vec{r},\vec{r}')\, e^{i\vec{p}\cdot(\vec{r}'-\vec{r})/\hbar}\, d\vec{r}$$

Normalization $\int n(\vec{p})\, d\vec{p} = N$

Homogeneous System $\rho(\vec{s} = \vec{r} - \vec{r}') = \frac{1}{V}\int n(\vec{p})\, e^{i\vec{p}\cdot\vec{s}/\hbar}\, d\vec{p} \xrightarrow{|\vec{s}|\to\infty} 0$ NO ODLRO



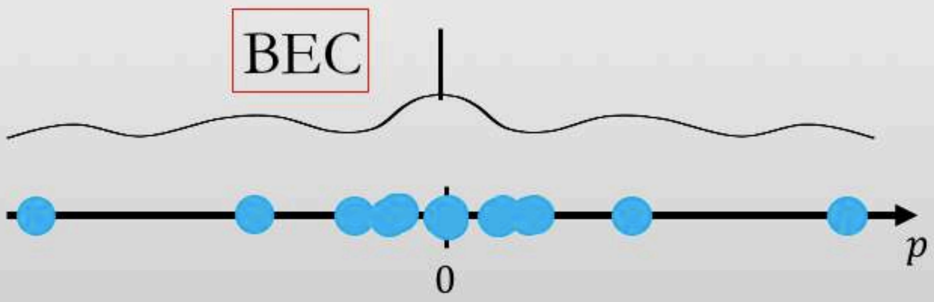


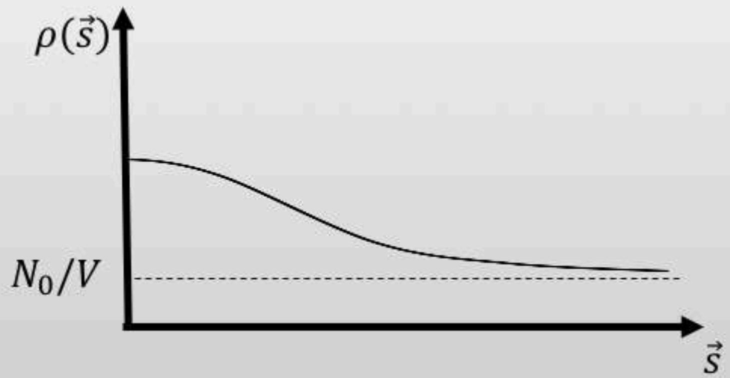
Momentum distribution

$$n(\vec{p}) = \frac{1}{(2\pi\hbar)^3} \int d\vec{r}' \int \rho(\vec{r},\vec{r}')\, e^{i\vec{p}\cdot(\vec{r}'-\vec{r})/\hbar}\, d\vec{r}$$

Normalization $\int n(\vec{p})\, d\vec{p} = N$

Homogeneous System $\rho(\vec{s} = \vec{r} - \vec{r}') = \frac{1}{V} \int n(\vec{p})\, e^{i\vec{p}\cdot\vec{s}/\hbar}\, d\vec{p} \xrightarrow{|\vec{s}| \to \infty} \frac{\lambda_0}{V}$ $\boxed{\text{ODLRO}}$





Outline

- Off-Diagonal Long-Range Order (ODLRO)
- ODLRO in 1D: Introduction to the Lieb-Liniger Model

  E.H. Lieb, W. Liniger, *Phys. Rev., 130 (1963) 1605*

  C.N. Yang, C.P. Yang, *J. Math. Phys., 10 (1969) 1115*
- One-body density matrix for Lieb-Liniger bosons
- One-body density matrix for Lieb-Liniger anyons (including 1D hard-core anyons)
- Conclusions

Lieb-Liniger Model

$\hbar = 2m = 1$

$$H = -\sum_{i=1}^{N} \frac{\partial^2}{\partial z_i^2} + 2c \sum_{1 \le i < j \le N} \delta(z_i - z_j)$$

$$\boxed{\gamma \equiv \frac{c}{n_{1D}}}$$ Dimensionless coupling

Low density ⇒ Tonks-Girardeau Gas ($\gamma \to \infty$) , $\chi_N \Big|_{z_i = z_j} = 0$ M. Girardeau, *J. Math. Phys., 1 (1960) 516*

PBC: $\chi_N(z_1 + L, \dots, z_N) = \chi_N(z_1, \dots, z_N)$

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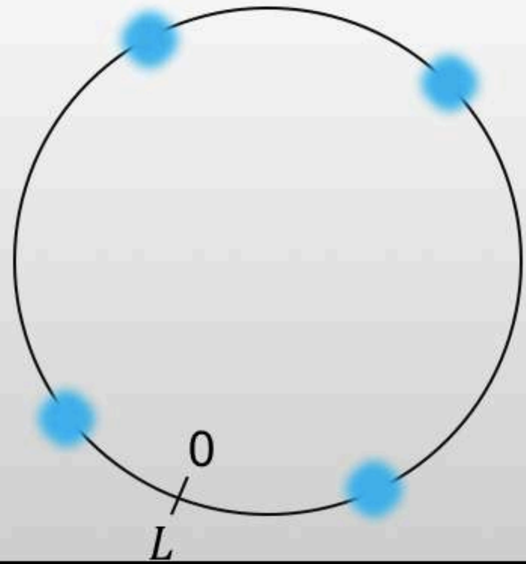

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PBC: $\chi_N(z_1 + L, \dots, z_N) = \chi_N(z_1, \dots, z_N)$

$$\lambda_j = \frac{2\pi}{L}\left(j - \frac{N+1}{2}\right) + \frac{2}{L}\sum_{k=1}^{N} \arctan\left(\frac{\lambda_j - \lambda_k}{c}\right), \qquad j = 1, \dots, N$$

$$\chi_N(z_1, \dots, z_N) = \mathcal{N} \det\left(e^{i\lambda_j z_m}\right) \prod_{n<l} [\lambda_l - \lambda_n - ic\, sign(z_l - z_n)]$$

Bethe ansatz solution

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Predictions for $\mathcal{C}$

$$\frac{n(p=0)}{L} \propto N^{\mathcal{C}}$$

$$n(p) = \frac{L}{2\pi}\int_0^L \rho(z)\, e^{ipz}\, dz$$

Large distance $\rho(z \gg 1)$ $\Longrightarrow$ Small momenta $n(p \approx 0)$

Luttinger Liquid prediction: $\rho(z \to \infty) \propto z^{-1/2K}$

$$K(\gamma) = \frac{v_F}{s(\gamma)}$$

$$p_{min} \approx \frac{2\pi}{L} \propto N^{-1} \qquad n_{1D} = \frac{N}{L}$$

$$\frac{n(p \to 0)}{L} \propto \frac{1}{p^{1-1/2K}} \propto N^{1-1/2K}$$

F.D.M. Haldane, *Phys. Rev. Lett., 47 (1981) 1840*
T. Giamarchi, *Quantum Physics in One Dimension*

Predictions for $\mathcal{C}(\kappa)$

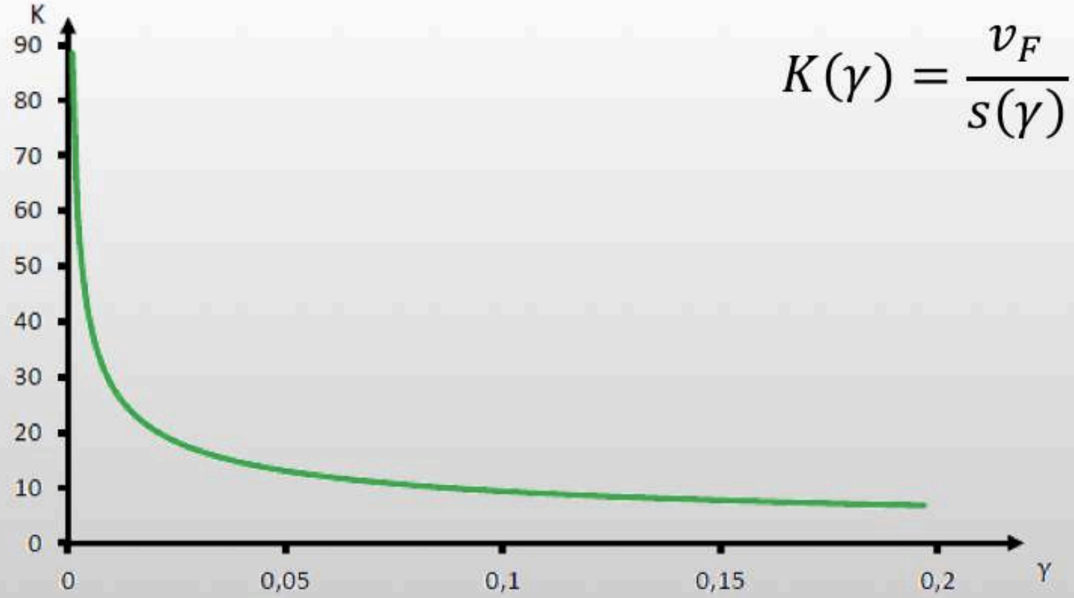
$$\frac{n(p=0)}{L} \propto N^{\mathcal{C}}$$

$$n(p) = \frac{L}{2\pi}\int_0^L \rho(z)\, e^{ipz}\, dz$$

Large distance $\rho(z \gg 1)$ $\Longrightarrow$ Small momenta $n(p \approx 0)$

$$\mathcal{C}(\gamma) = 1 - \frac{1}{2K}$$

Deviation from ODLRO



$$K(\gamma) = \frac{v_F}{s(\gamma)}$$

F.D.M. Haldane, *Phys. Rev. Lett., 47 (1981) 1840*
T. Giamarchi, *Quantum Physics in One Dimension*

Lieb-Liniger Bosons

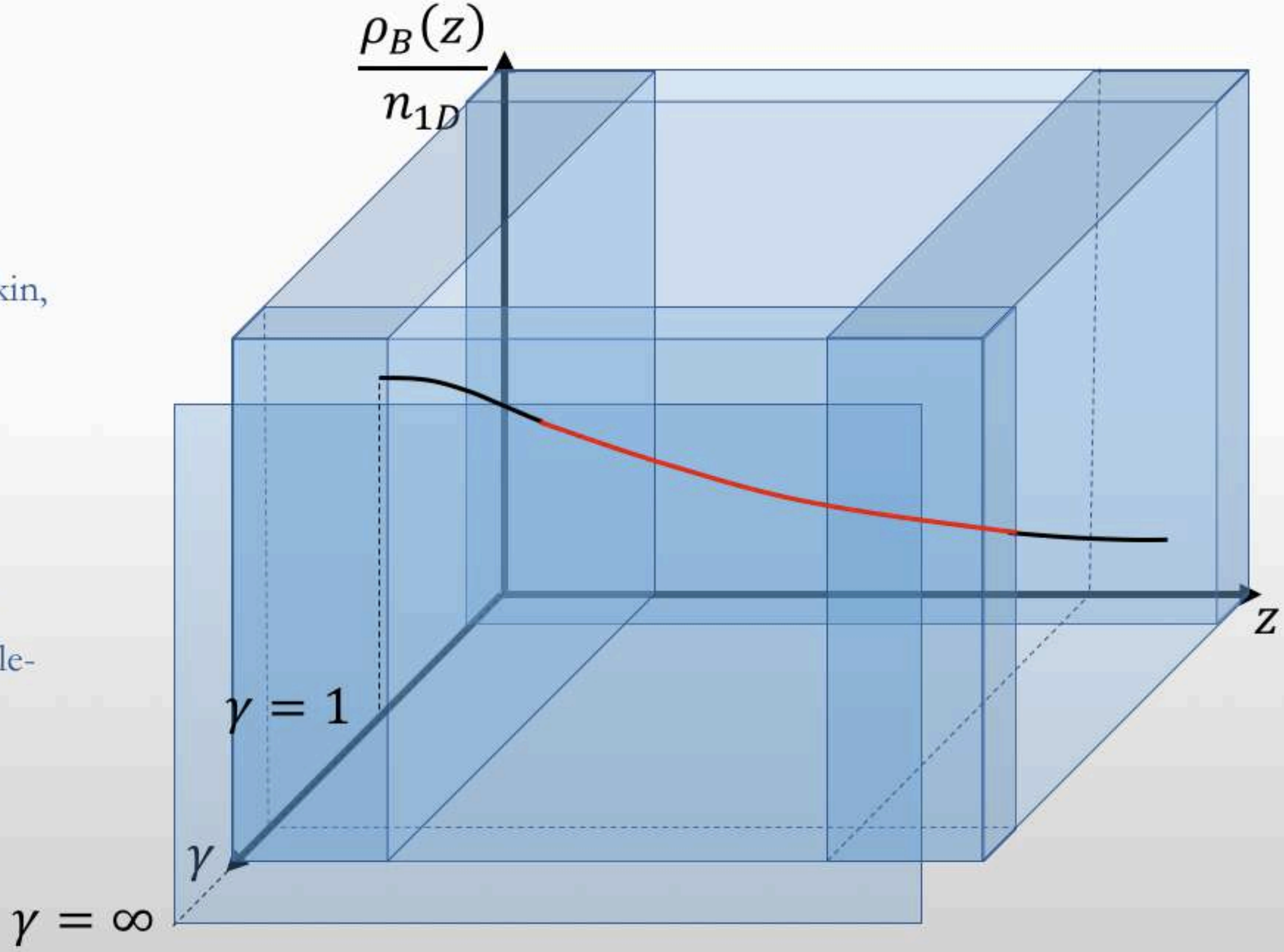
Small γ: C. Mora, Y. Castin,
Phys. Rev. A, 67 (2003) 053615.

Large γ: M. Jimbo, T. Miwa,
Phys. Rev. D, 24 (1981) 3169.
P.J. Forrester, N.E. Frankel, M.I. Makin,
Phys. Rev. A, 74 (2006) 043614.

TG limit: A. Lenard,
J. Math. Phys., 5 (1964) 930.

Small z: M. Olshanii, V. Dunjko,
Phys. Rev. Lett., 91 (2003) 090401.

Large z: A. Shashi, M. Panfil, J.-S. Caux, I. Alexander,
Phys. Rev. B, 85 (2012) 155136.

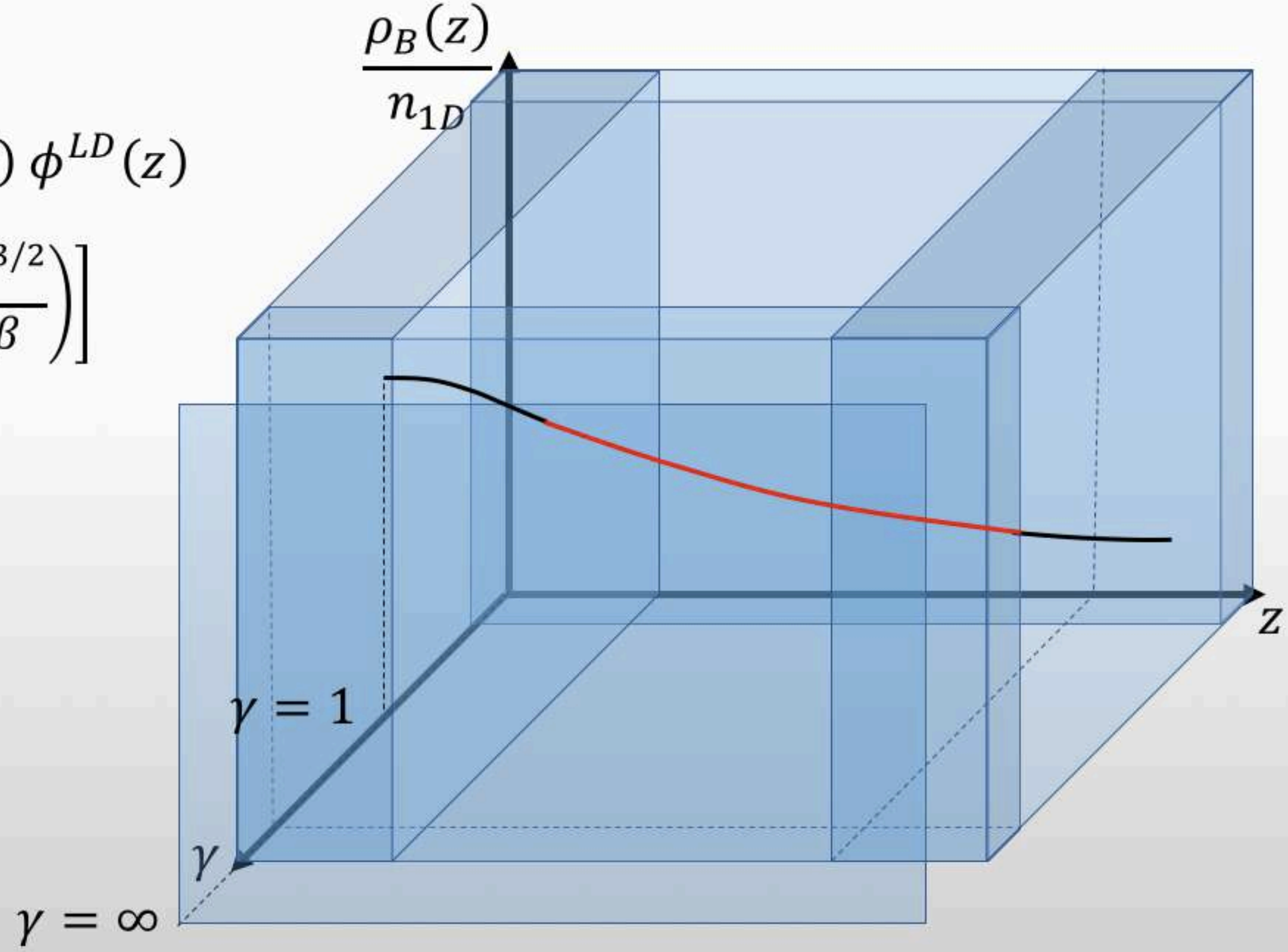


Lieb-Liniger Bosons

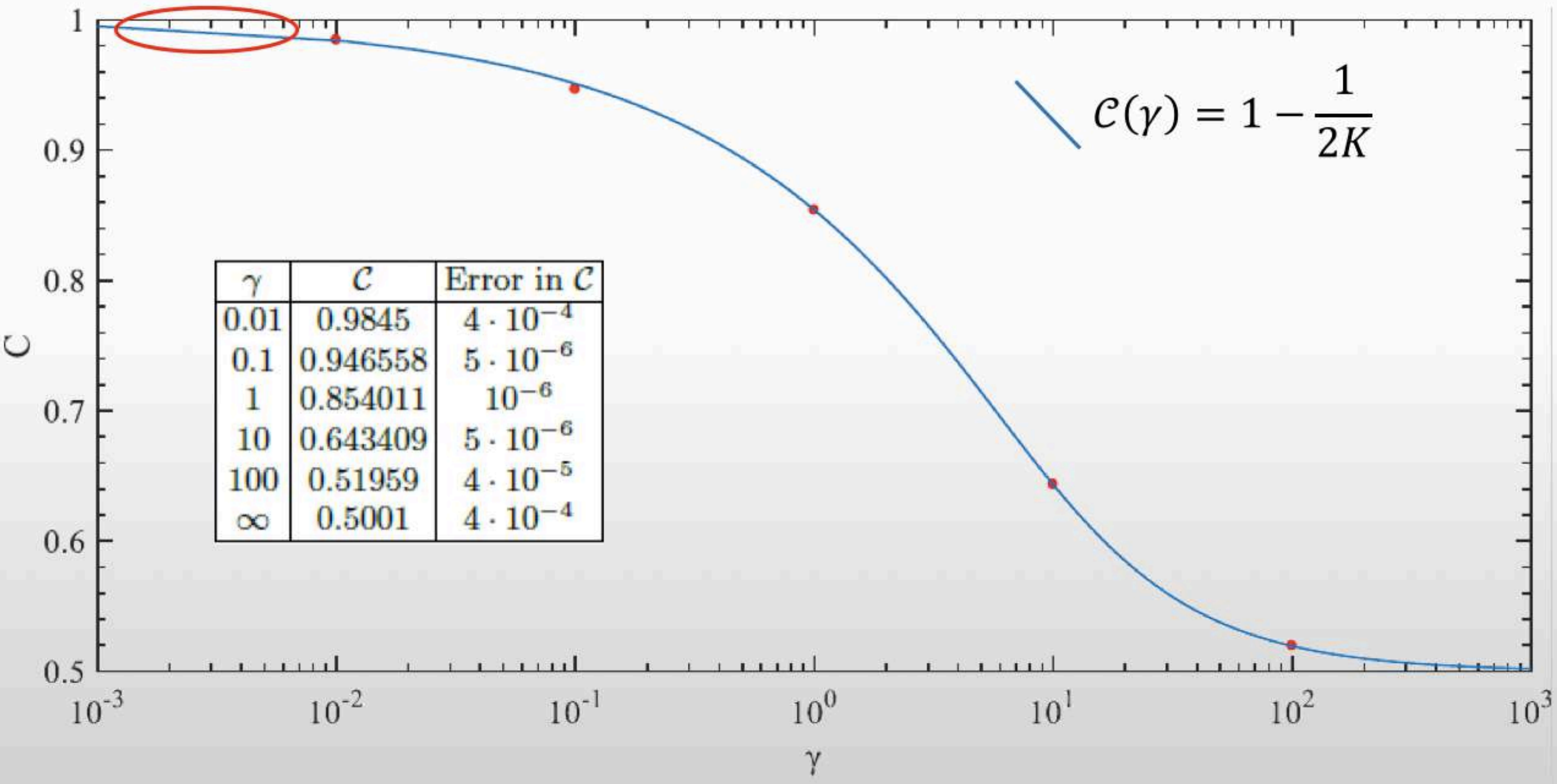
$$\rho_B(z) = \rho_B^{SD}(z)\,\phi^{SD}(z) + \rho_B^{LD}(z)\,\phi^{LD}(z)$$

$$\phi^{SD}(z) = \left[1 - tgh\left(\frac{z}{\alpha}\right)\right]\left[1 - tgh\left(\frac{z^{3/2}}{\beta}\right)\right]$$

$$\phi^{LD}(z) = tgh\left(\frac{z}{\eta}\right) tgh\left(\frac{z}{\omega}\right)$$



Lieb-Liniger Bosons



γ	$\mathcal{C}$	Error in $\mathcal{C}$
0.01	0.9845	$4 \cdot 10^{-4}$
0.1	0.946558	$5 \cdot 10^{-6}$
1	0.854011	10^{-6}
10	0.643409	$5 \cdot 10^{-6}$
100	0.51959	$4 \cdot 10^{-5}$
∞	0.5001	$4 \cdot 10^{-4}$

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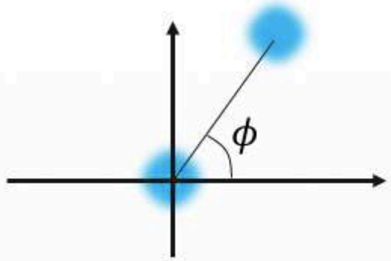
Lieb-Liniger Model with anyons



$$H = -\sum_{i=1}^{N} \frac{\partial^2}{\partial z_i^2} + 2c \sum_{1\le i<j\le N} \delta(z_i - z_j)$$

$$\chi_N^\kappa(z_1, \dots, z_i, z_{i+1}, \dots, z_N) = e^{-i\pi\kappa\, sign(z_i - z_{i+1})} \chi_N^\kappa(z_1, \dots, z_{i+1}, z_i, \dots, z_N)$$

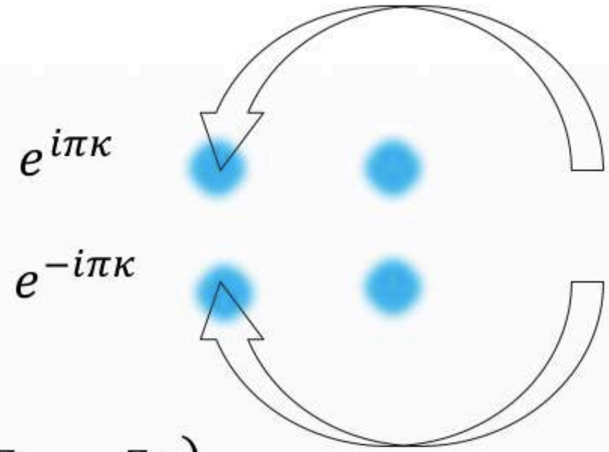
$$\chi_N^\kappa(z_1, \dots, z_i, \dots, z_j, \dots, z_N) = e^{-i\pi\kappa\left[\sum_{k=i+1}^{j} sign(z_i - z_k) - \sum_{k=i+1}^{j-1} sign(z_j - z_k)\right]} \cdot$$

$$\cdot\, \chi_N^\kappa(z_1, \dots, z_j, \dots, z_i, \dots, z_N)$$

κ = Statistical parameter

A. Kundu,
Phys. Rev. Lett., 83 (1999) 1275

Lieb-Liniger Model with anyons



$$H = -\sum_{i=1}^{N} \frac{\partial^2}{\partial z_i^2} + 2c \sum_{1 \le i < j \le N} \delta\left(z_i - z_j\right)$$

$$\chi_N^\kappa(z_1, \dots, z_i, z_{i+1}, \dots, z_N) = e^{-i\pi\kappa\, sign(z_i - z_{i+1})} \chi_N^\kappa(z_1, \dots, z_{i+1}, z_i, \dots, z_N)$$

$$\chi_N^\kappa\left(z_1, \dots, z_i, \dots, z_j, \dots, z_N\right) = e^{-i\pi\kappa\left[\sum_{k=i+1}^{j} sign(z_i - z_k) - \sum_{k=i+1}^{j-1} sign(z_j - z_k)\right]} \cdot$$

$$\cdot\, \chi_N^\kappa\left(z_1, \dots, z_j, \dots, z_i, \dots, z_N\right)$$

$$\kappa = \text{Statistical parameter} \begin{cases} 0 & \text{Bosons} \\ 1 & \text{Fermions,} \end{cases}$$

A. Kundu,
Phys. Rev. Lett., 83 (1999) 1275

Lieb-Liniger Model with anyons

Twisted BC: $\chi_N^\kappa(0, x_2, \dots) = e^{i\pi\kappa(N-1)}\chi_N^\kappa(L, x_2, \dots) \longleftrightarrow$ Periodic BC: $\chi_N^0(0, x_2, \dots) = \chi_N^0(L, x_2, \dots)$

O. I. Pâţu, V. E. Korepin, D. V. Averin, *J. Phys. A, 40 (2007) 14963*

$$\lambda_j = \frac{2\pi}{L}\left(j - \frac{N+1}{2}\right) + \frac{2}{L}\sum_{k=1}^{N} \arctan\left(\frac{\lambda_j - \lambda_k}{c'}\right), \qquad j = 1, \dots, N \qquad \boxed{c' = \frac{c}{\cos(\pi\kappa/2)}} > 0$$

$$\chi_N^\kappa(z_1, \dots, z_N) = \mathcal{N} \exp\left(i\frac{\pi\kappa}{2}\sum\nolimits_{j<k} sign\left(z_j - z_k\right)\right)\det\left(e^{i\lambda_j z_m}\right) \cdot$$

$$\cdot \prod\nolimits_{n<l}[\lambda_l - \lambda_n - ic' sign(z_l - z_n)]$$

M. T. Batchelor, X.-W. Guan, N. Oelkers, *Phys. Rev. Lett., 96 (2006) 210402*
M. T. Batchelor, X.-W. Guan, J.-S. He, *J. Stat. Mech., (2007) P03007*

Off-Diagonal Long-Range Order

$$\rho_A(z,z') = N\int_0^L dz_2 \ldots dz_N \, [\chi_N^\kappa(z,z_2,\ldots,z_N)]^* \, \chi_N^\kappa(z',z_2,\ldots,z_N)$$

$$\chi_N^\kappa(z+L,z_2,\ldots,z_N) = e^{-i\pi\kappa(N-1)}\chi_N^\kappa(z,z_2,\ldots,z_N)$$

$$\rho_A(z+L) = e^{i\pi(1-\kappa)(N-1)}\rho_A(z)$$

$$\boxed{\text{Homogeneous System \& Periodicity in OBDM}} \Longrightarrow \boxed{\kappa = \frac{m}{n} \in \mathbb{Q}}$$

$$n,m = 0,1,2,\ldots,m \le n$$

$$\boxed{\lambda_0 \propto N^{\mathcal{C}(\kappa)} \propto \frac{n_A(p = -p_F\kappa)}{L}}$$

Predictions for $\mathcal{C}(\kappa)$

$n_A(p=0) \propto N^{\mathcal{C}(\kappa)}$

$$n_A(p) = \frac{L}{2\pi}\int_0^L \rho_A(z)\, e^{ipz}\, dz$$

Large distance $\rho_A(z \gg 1)$ $\Longrightarrow$ "Small" momenta $n_A(p \approx -p_F\kappa)$

$\boxed{\kappa = 0}$ $\quad \mathcal{C}(\kappa=0,\gamma) = 1 - \frac{1}{2K}$ $\qquad K(\gamma) = \frac{v_F}{s(\gamma)}$

$\boxed{\kappa \text{ generic}}$ $\quad \frac{n_A(p)}{L} \propto \frac{1}{(p+p_F\kappa)^{1-\frac{1}{2K}-\frac{K\kappa^2}{2}}}$ $\Longrightarrow$ $\boxed{\mathcal{C}(\kappa,\gamma) = 1 - \frac{1}{2K} - \frac{K\kappa^2}{2}}$

P. Calabrese, M. Mintchev, *Phys. Rev. B, 75 (2007) 233104*

Hard-Core Anyons $\gamma \to \infty$

$$\chi_N^{\kappa}(z_1, \dots, z_N) = \left[\prod_{1 \le i < j \le N} A(z_j - z_i)\right] \chi_N^{1}(z_1, \dots, z_N) \qquad \kappa = \frac{m}{n} \in \mathbb{Q}$$

Anyon – Fermi mapping: $A(z_j - z_i) = \left[\theta(z_j - z_i) + \theta(z_i - z_j) e^{i\pi(1-\kappa)}\right]$ $\qquad \theta(0) = 0$

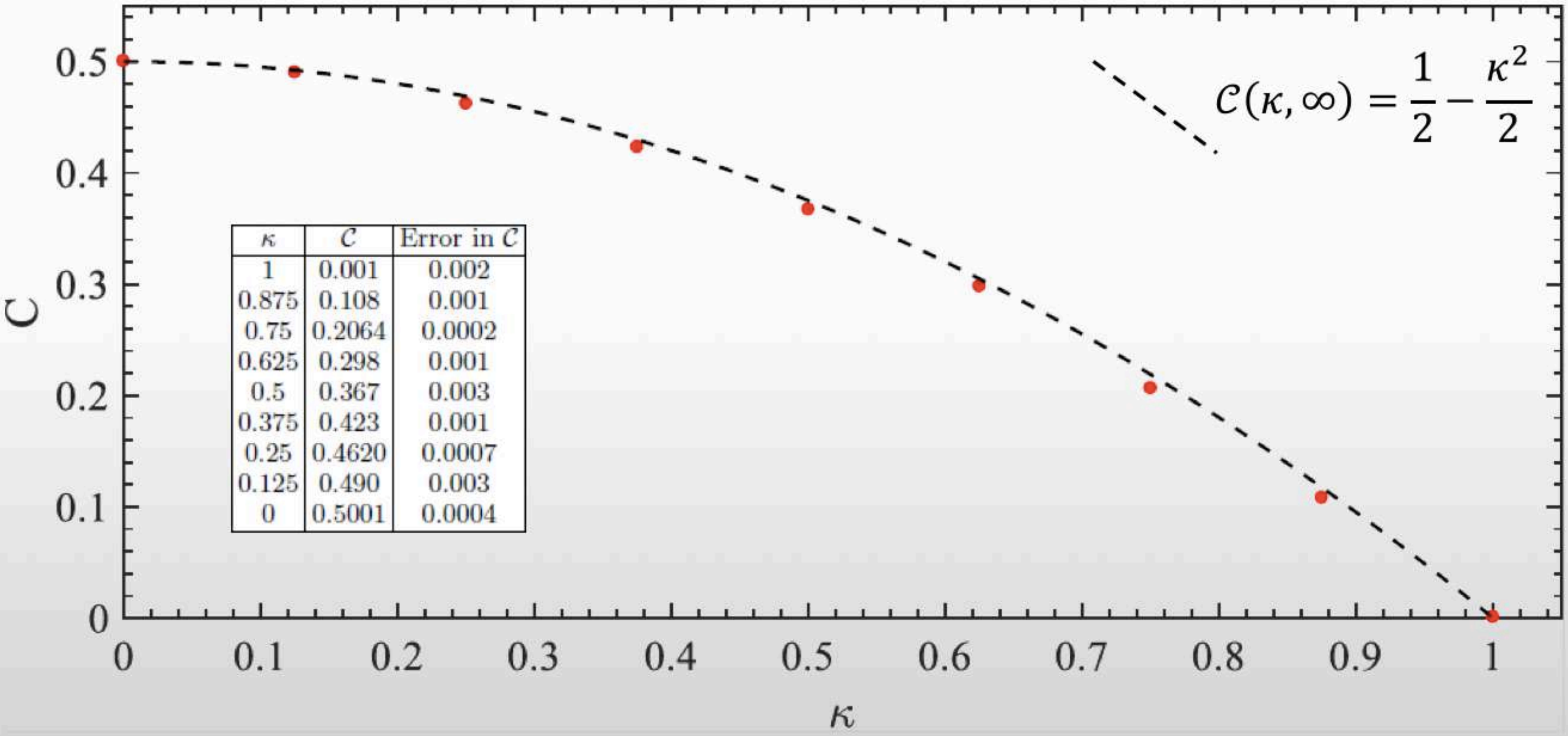
$\boxed{\kappa = 0}$ Boson – Fermi mapping: $A(z_j - z_i) = sign(z_j - z_i)$

$$\rho_A(t) = \det\left[\frac{2}{\pi}\int_0^{2\pi} d\tau\, e^{i(j-l)\tau} A(\tau - t) \sin\left(\frac{\tau - t}{2}\right) \sin\left(\frac{\tau}{2}\right)\right]_{j,l=1,\dots,N-1} \qquad t \equiv \frac{2\pi x}{L}$$

M.D. Girardeau, *Phys. Rev. Lett., 97 (2006) 210401*

R. Santachiara, P. Calabrese, *J. Stat. Mech., (2008) P06005*

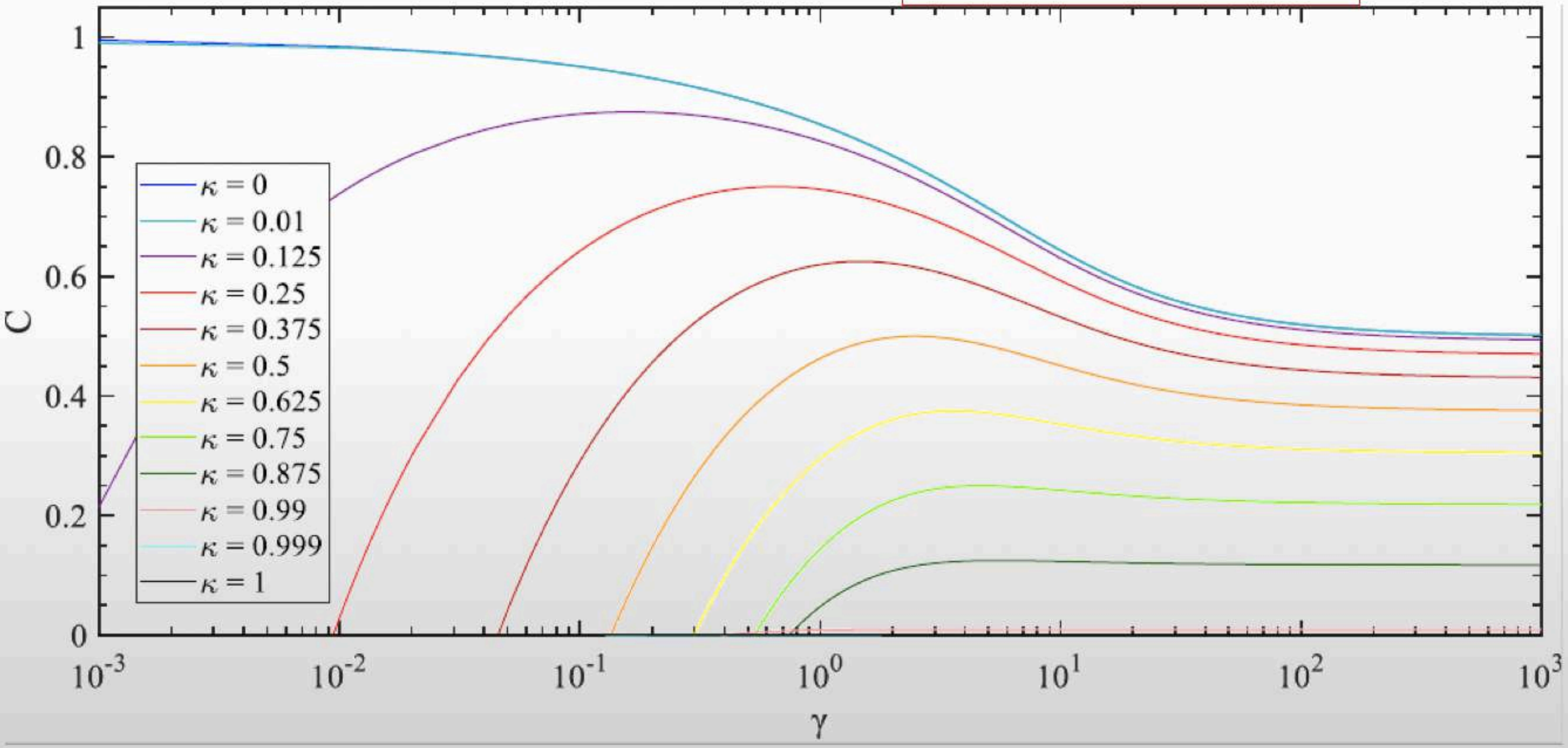
Hard-Core Anyons $\mathcal{C}(\kappa, \gamma \to \infty)$



κ	$\mathcal{C}$	Error in $\mathcal{C}$
1	0.001	0.002
0.875	0.108	0.001
0.75	0.2064	0.0002
0.625	0.298	0.001
0.5	0.367	0.003
0.375	0.423	0.001
0.25	0.4620	0.0007
0.125	0.490	0.003
0	0.5001	0.0004

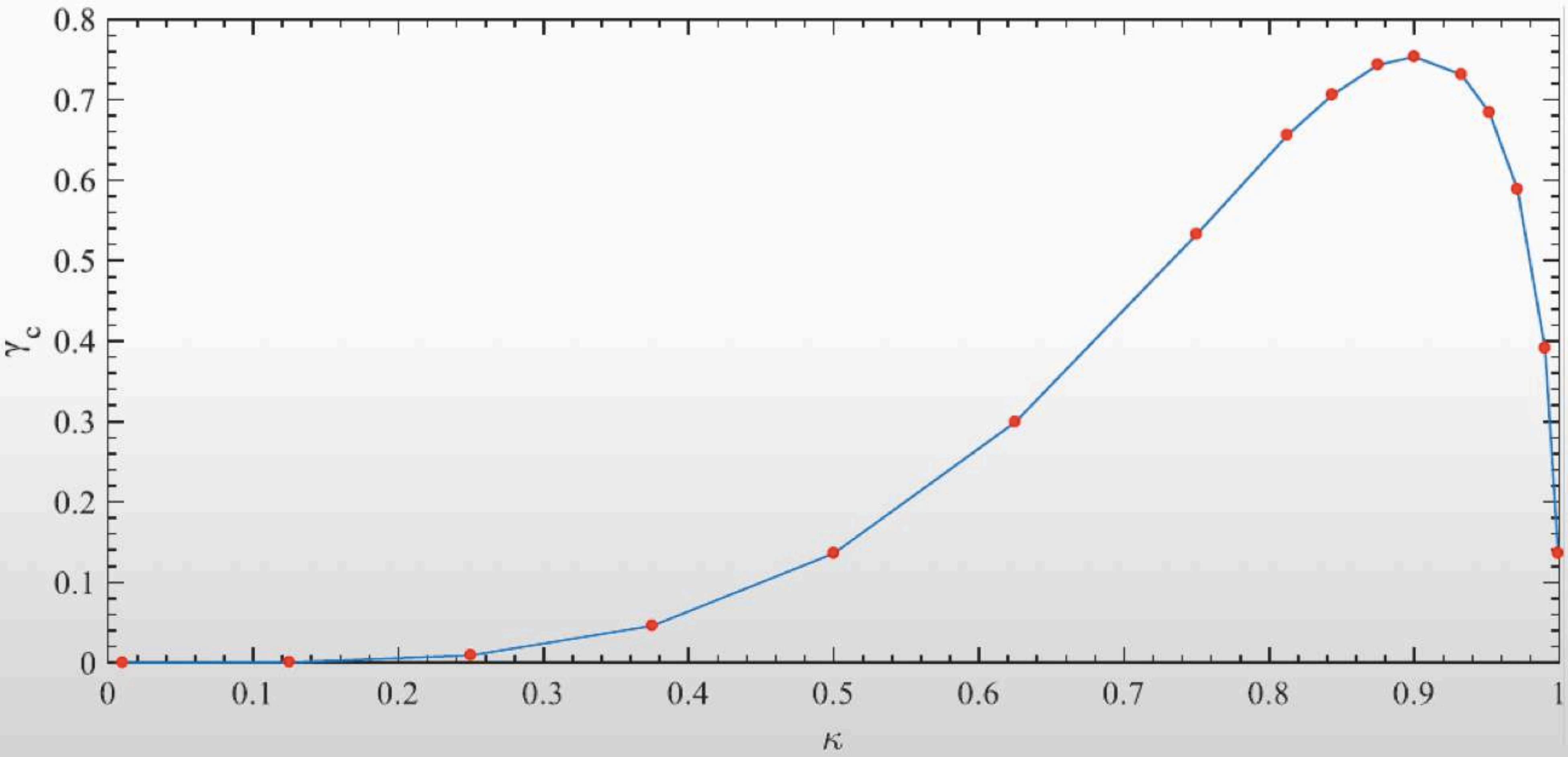
Predictions for $\mathcal{C}(\kappa, \gamma)$

$$\mathcal{C}(\kappa, \gamma) = 1 - \frac{1}{2K} - \frac{K\kappa^2}{2}$$



Critical Coupling

$$\mathcal{C}(\kappa, \gamma_c) = 0$$



Outline

- Off-Diagonal Long-Range Order (ODLRO)

- ODLRO in 1D: Introduction to the Lieb-Liniger Model

- One-body density matrix for Lieb-Liniger bosons

- Conclusions

Conclusions and Outlook

- ODLRO in terms of occupation numbers

- Quantify deviations from ODLRO in 1D Systems

- Bosonization and Harmonic Fluid Approach as check

- Inhomogeneous LL bosons (finite coupling), Finite Temperature, Experimental Results …

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ma per seguir virtute e canoscenza

The Abdus Salam International Centre for Theoretical Physics

ICTP

Scrambling and entanglements spreading in long range spin chains

Silvia Pappalardi

Trieste Junior Quantum Days

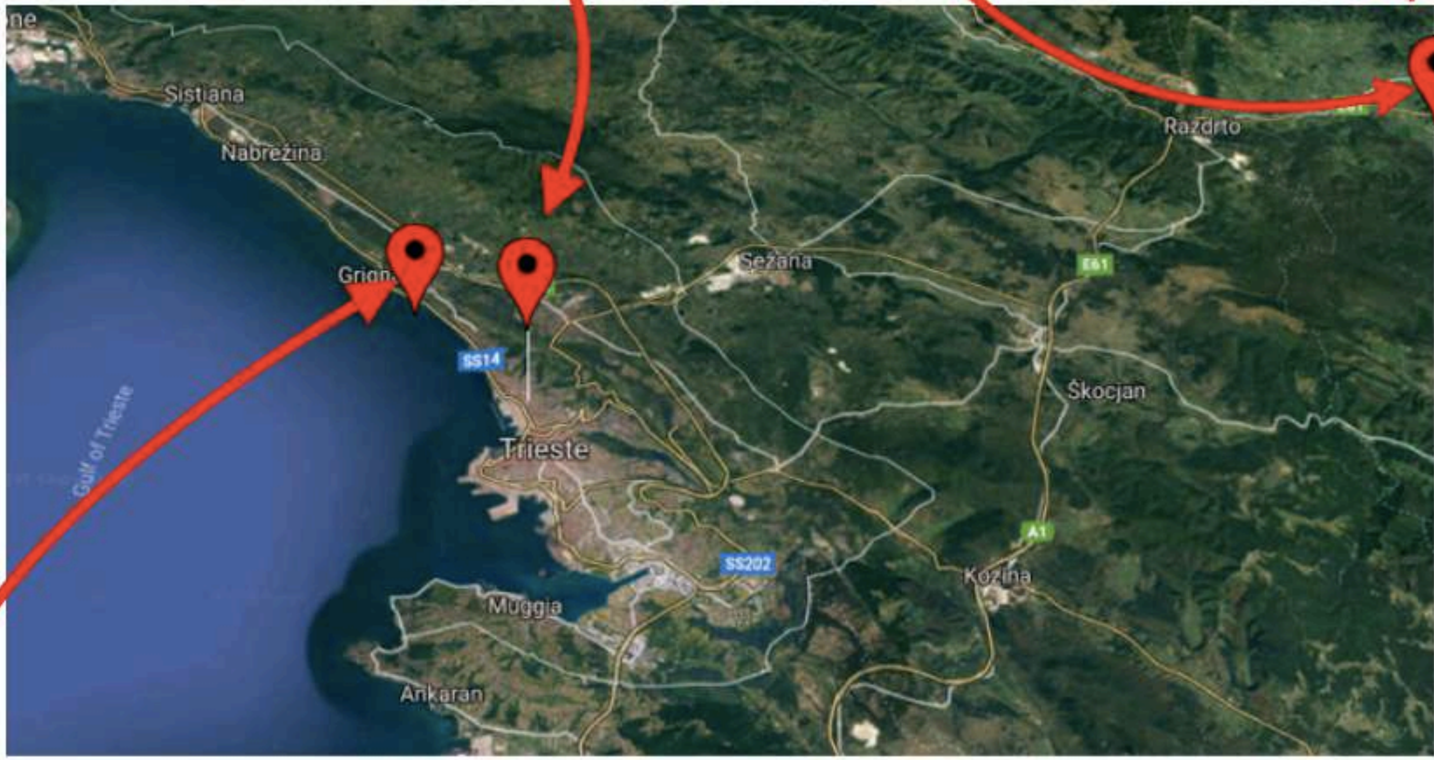
A glance in research: where we stand and future challenges

Alessandro Silva

ma per seguir virtute e canoscenza
SISSA

University of Ljubljana
Faculty of Mathematics and Physics

Bojan Zunkovič



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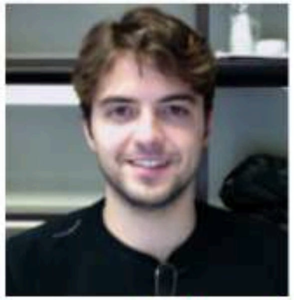


Rosario Fazio Angelo Russomanno Fernando Iemini

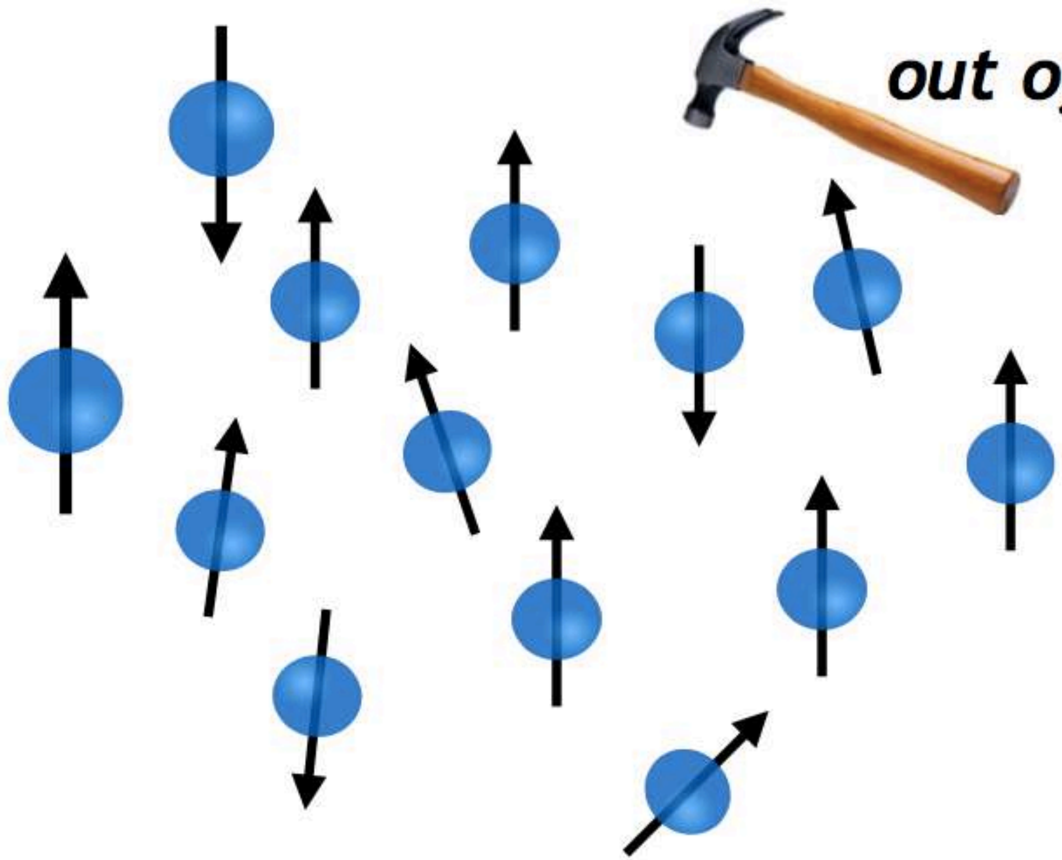
Many-body systems dynamics

$$|\psi_{tot}\rangle \in \bigotimes_{i=1}^{n} \mathcal{H}_i \quad \text{dim. } 2^n$$

How does information propagates?

$\hat{U}(t) = e^{i\hat{H}t}$ unitary evolution $\qquad |\psi_{tot}(t)\rangle = e^{i\hat{H}t}|\psi_{tot}\rangle$

out of equilibrium



1. "spreading of quantum information across the system"

quantum chaos from the ~~semiclassical~~ limit

$\longrightarrow$ OTOC correlators

Scrambling and entanglements spreading in long range spin chains

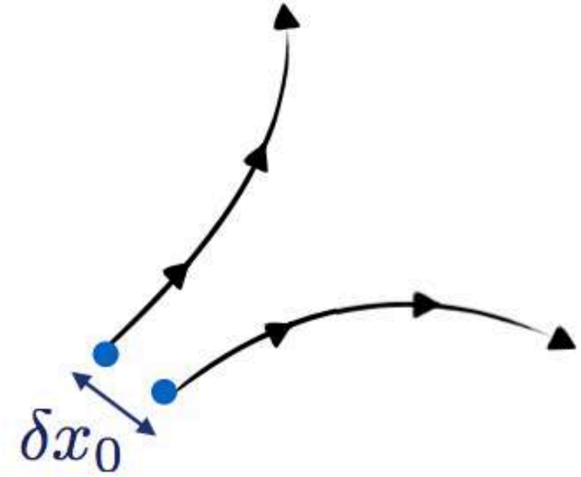
2. $\neq$

3.

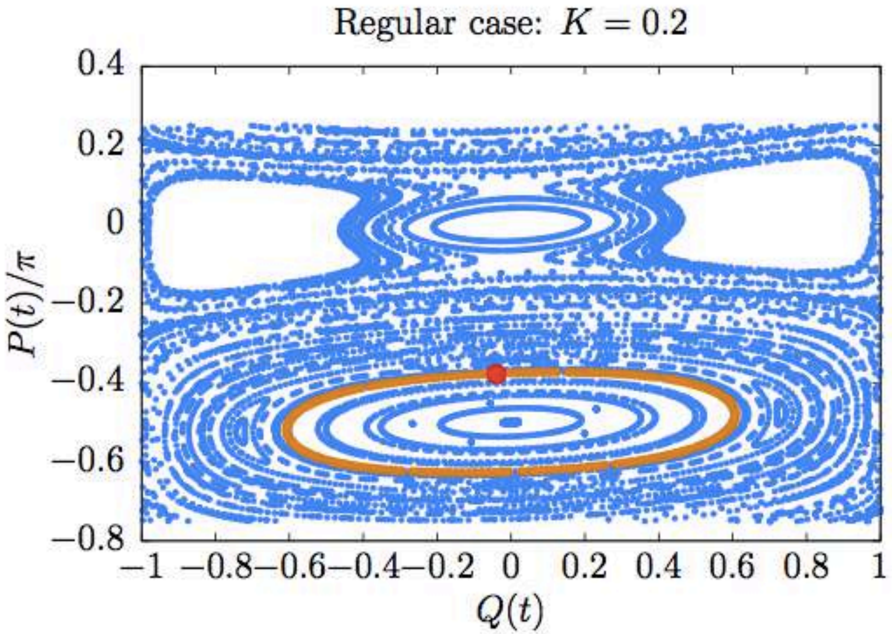
Classical chaos

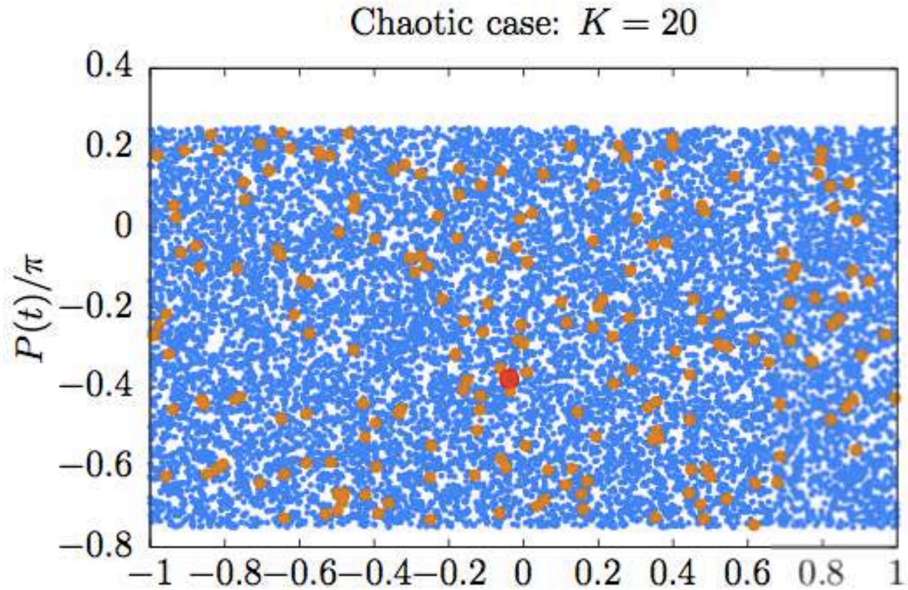
"exponential deviation of trajectories"

$$\left|\frac{\partial x(t)}{\partial x_0}\right| \sim e^{\lambda t}$$ Lyapunov exponent



Poincaré section





MV Berry - AIP Conference proceedings, 1978

Scrambling 1.

Quantum chaos

"hypersensitivity to perturbations of H"

On states:

1984 Peres

$$m(t) = \langle \psi_0 | e^{i(\hat{H}+\delta\hat{B})t}\, e^{-i\hat{H}t} | \psi_0 \rangle$$

Loschmidt echo

2001 Levstein-Jalambert-Pastawski

In operator space:

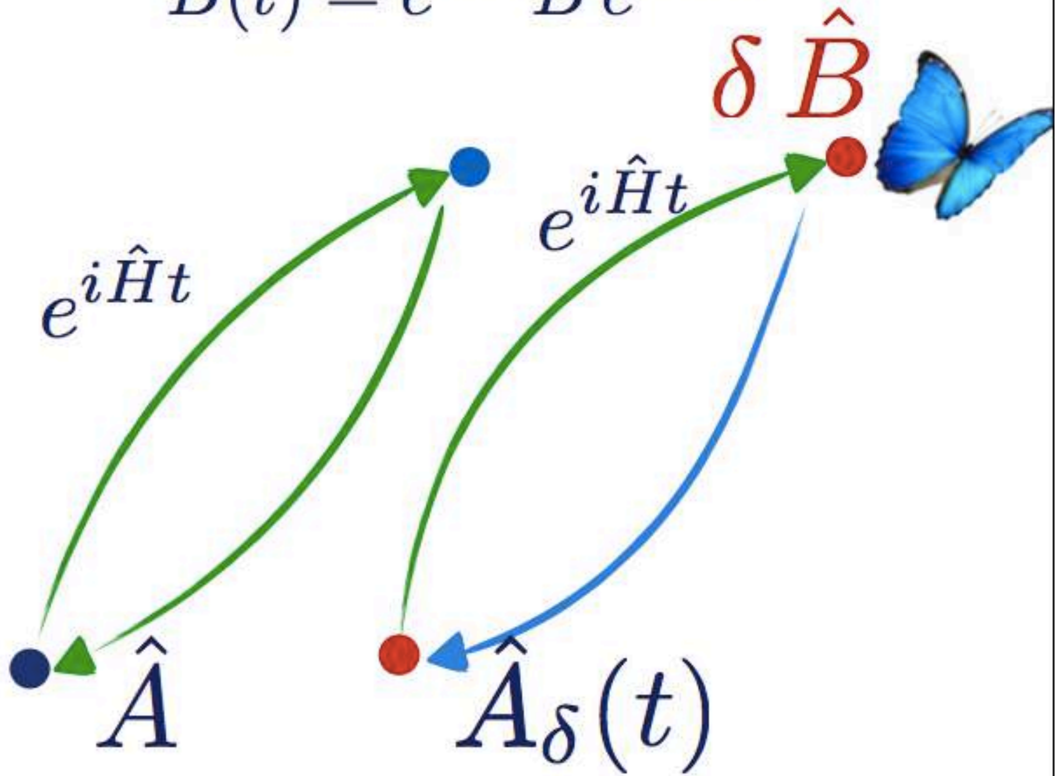
$$\hat{B}(t) = e^{i\hat{H}t}\,\hat{B}\,e^{-i\hat{H}t}$$

$$\hat{A}_\delta(t) = e^{i\delta\hat{B}(t)}\,\hat{A}\,e^{i\delta\hat{B}(t)}$$

$$\simeq \hat{A} + i\delta\,[\hat{B}(t), \hat{A}]$$

$$\langle (\hat{A}_\delta(t) - \hat{A})^2 \rangle =$$

$$-\delta^2\,\langle [\hat{B}(t), \hat{A}]^2 \rangle$$



The square commutator

Larkin and Ovchinnikov. 1969 SOVIET PHYSICS JETP

QUASICLASSICAL METHOD IN THE THEORY OF SUPERCONDUCTIVITY

$$C(t) = -\langle [\hat{x}(t), \hat{p}(0)]^2 \rangle \xrightarrow{\text{canonical quantization}} \hbar^2 \{x(t), p_0\}^2 = \hbar^2 \left(\frac{\partial x(t)}{\partial x_0} \right)^2$$

Larkin, Ovchinnikov - Sov Phys JETP, 1969

Kitaev. 2015

- many-body system
- to generic operators
- SYK model

(Majorana fermions: all to all random interaction)

$$C(t) \sim e^{\lambda_Q t}$$

What is scrambling?

$$[\hat{B}, \hat{A}] = 0$$

$$\hat{B}(t) = e^{i\hat{H}t}\hat{B}e^{-i\hat{H}t} = \hat{B} + it[\hat{H}, \hat{B}] - \frac{t^2}{2}[\hat{H}, [\hat{H}, \hat{B}]] + \mathcal{O}(t^3)$$

$$[\hat{B}(t), \hat{A}] \neq 0$$

scrambling: non-commutativity induced by the dynamics!

$$\boxed{C(t) = -\langle[\hat{B}(t), \hat{A}\,]^2\rangle_\beta} = \langle \hat{B}(t)\,\hat{A}\,\hat{A}\,\hat{B}(t)\rangle + \langle \hat{A}\,\hat{B}(t)\,\hat{B}(t)\,\hat{A}\,\rangle$$
$$-\langle \hat{B}(t)\,\hat{A}\,\hat{B}(t)\,\hat{A}\rangle - \langle \hat{A}\,\hat{B}(t)\,\hat{A}\,\hat{B}(t)\,\rangle$$

"out-of-time ordered correlators"
OTOC

expectation for a "chaotic quantum system"

$$\sim \epsilon\, e^{\lambda_Q t} \quad 0 \leq \lambda_Q \leq 2\pi T$$

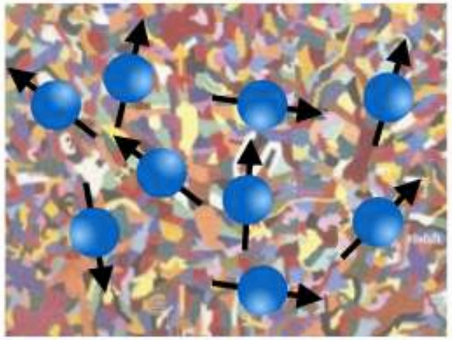
underlying classical limit

Maldacena, Shenker, Stanford - Journal of High Energy Physics, 2016

Scaffidi, Altman - arXiv:1711.04768, 2017
Cotler, Ding, Penington - arXiv:1704.02979, 2017

Non-exponential behavior of $C(t)$

Disordered systems (+ interactions)



- extended (thermal) phase
- MBL phase $C(t) \sim t^{\alpha}$

Chen, Zhou, Huse, Fradkin - Annalen der Physik, 2017

Short range on the lattice

- extensive operators $\hat{A} \equiv \sum_i \hat{\sigma}_i$
- lattice models
- local interactions

$$\left.\begin{array}{l}\text{extensive operators} \\ \text{lattice models} \\ \text{local interactions}\end{array}\right\} \quad c(t) \leq A\, t^{3d}$$

Kukuljan, Grozdanov, Prosen - Phys. Rev. B, 2017

!!! relevant in IQ

Hosur, Qi, Roberts, Yoshida - Journal of High Energy Physics, 2016

Scrambling 1. 

scrambling: non-commutativity of operators induced by the dynamics

square-commutator: introduced in quanto chaos goes exponentially:classical underlying

entanglements spreading 2.

Entanglement

*"**entanglement** is rather **the** characteristic trait of quantum mechanics."*

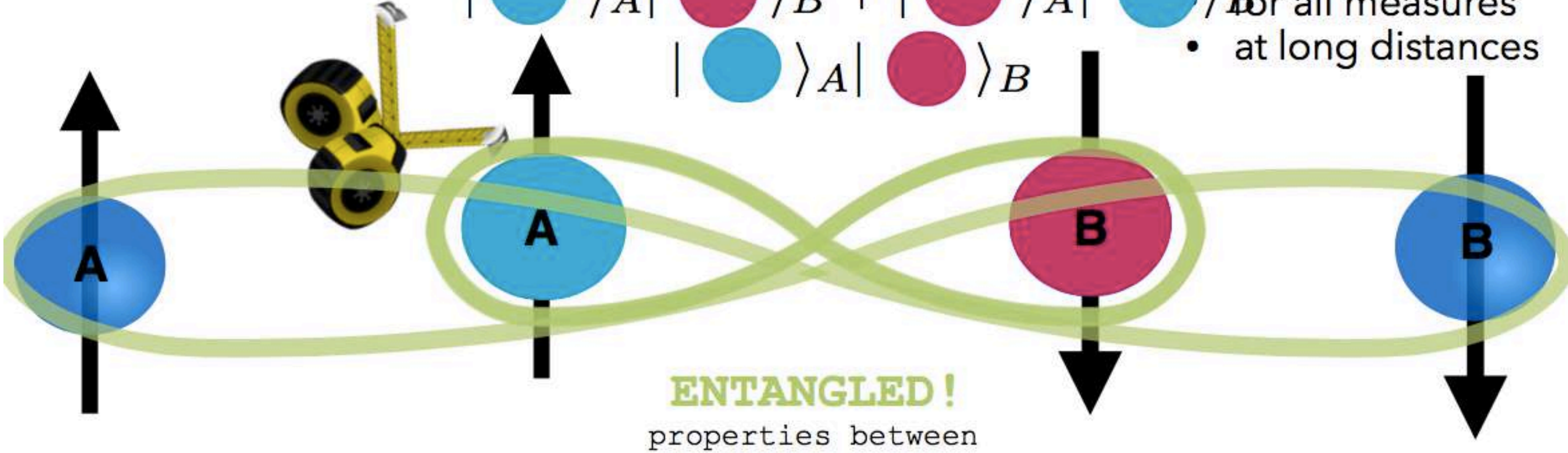
E. Schrödinger, 1935

QUANTUM WORLD: more than an object

$$|{\color{cyan}\bullet}\rangle_A|{\color{red}\bullet}\rangle_B + |{\color{red}\bullet}\rangle_A|{\color{cyan}\bullet}\rangle_B$$

$$|{\color{cyan}\bullet}\rangle_A|{\color{red}\bullet}\rangle_B$$

- for all measures
- at long distances



ENTANGLED!

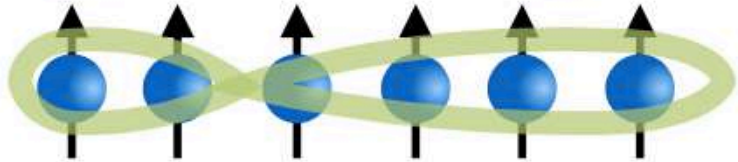
properties between the two systems are correlated

Entanglements dynamics

entanglement entropy

$$S_L(t) = -\mathrm{Tr}(\hat{\rho}_L \log \hat{\rho}_L)$$

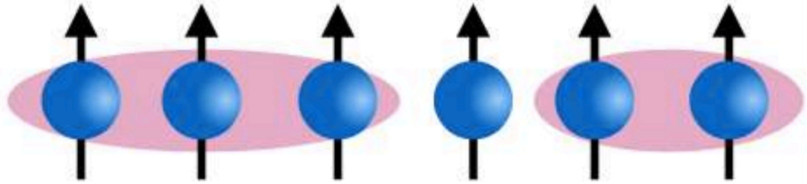
Bipartite entanglement



Quantum Fisher Information

$$f_Q(t)$$

Multipartite entanglement



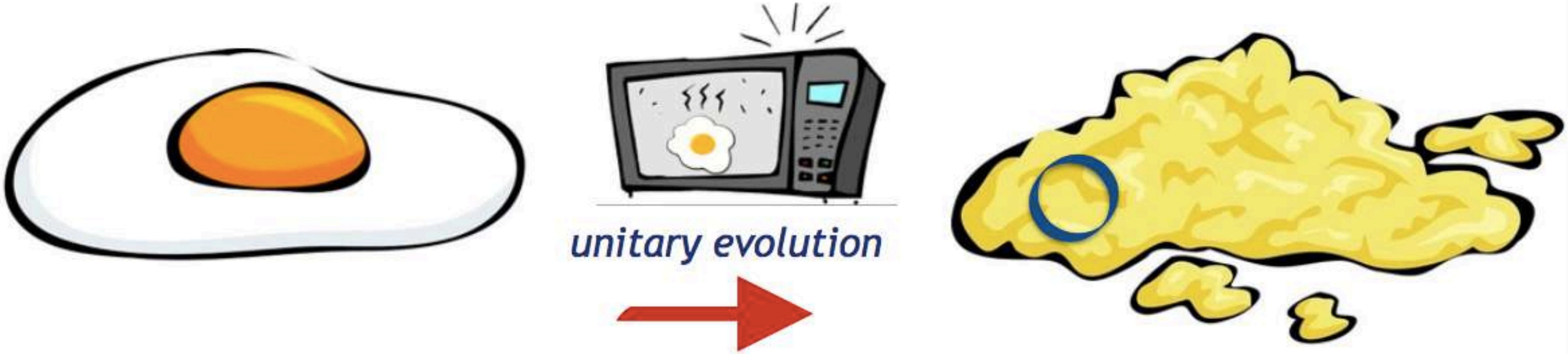
if $f_Q \sim N$
globally entangled state!

bound on the *size of the biggest entangled block*

?!

Scrambling and entanglement

spreading of information = entanglement dynamics



globally → pure state

locally → observables thermalize: initial conditions are lost

the information is hidden non-locally in the correlations between subsystems: entanglement

Scrambling 1. 

scrambling: non-commutativity of operators induced by the dynamics

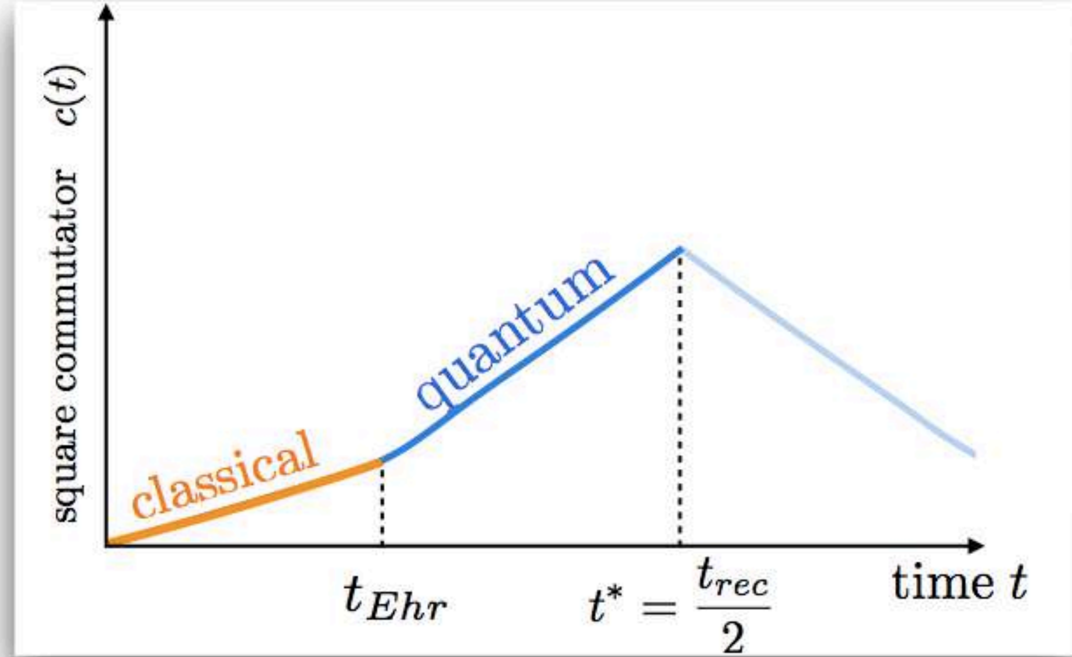
square-commutator: introduced in quanto chaos goes exponentially:classical underlying

entanglements spreading 2. 

globally information is conserved: hidden non-locally in entanglement

3. long range spin chains

life beyond semi-classics



chaotic systems

- exponential
- saturation
- $t_{rec} \sim e^N$

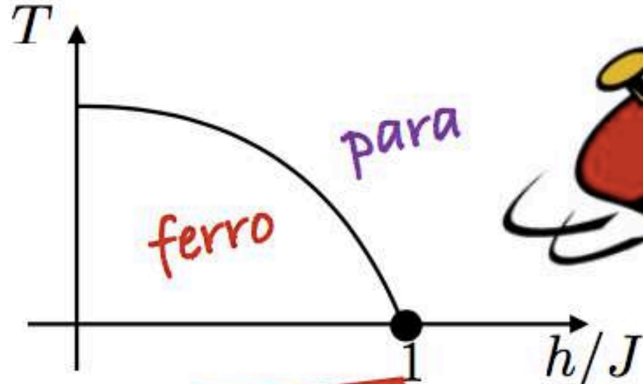
regular systems

- polynomial
- polynomial growth
- $t_{rec} \sim N$
- different from entanglement!

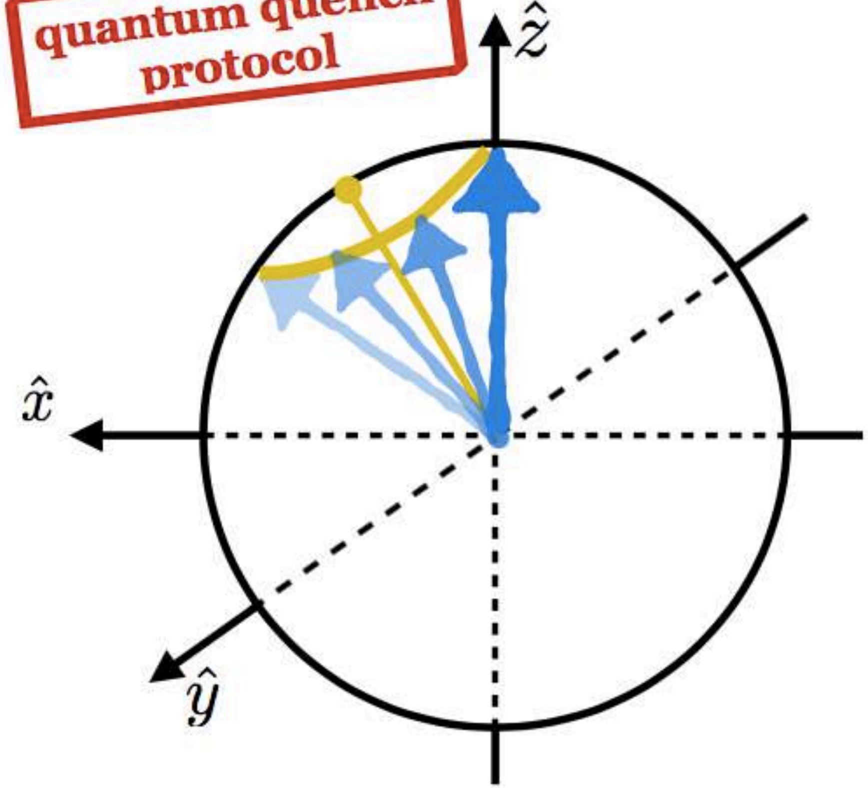
The model. Lipkin-Meshkov-Glick

Infinite range Ising model

$$H(h) = -\frac{2J}{N}\sum_{ij}^{N} S_i^z\, S_j^z - 2\cancel{h_f \sum_{i}^{N}} S_i^x$$



quantum quench protocol



- solvable
- $[\hat{S}^2, \hat{H}] = 0, \;\; \vec{S} = \sum_i \vec{S}_i$
- semiclassical limit $\hbar_{eff} \sim 1/N$
- initial state $|\psi_0\rangle = |\uparrow\uparrow \dots \uparrow\rangle$

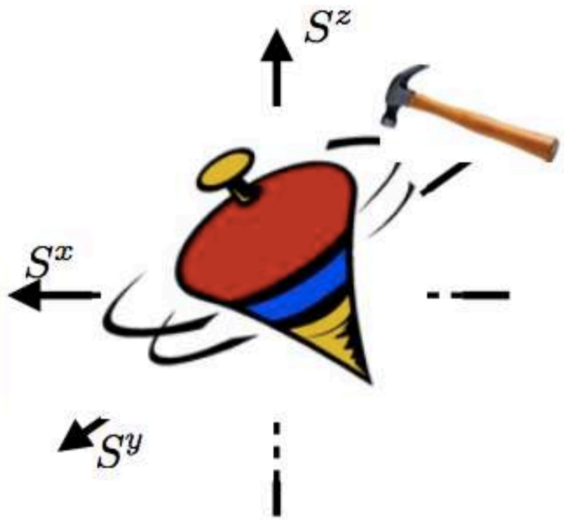
- long range $J_{ij} \sim \dfrac{1}{|i-j|^{\alpha}}$

Sciolla, Biroli - Phys. Rev. Lett., 2010

Lipkin, Meshkov, Glick - Nuclear Physics, 1965

The kicked top

$$\hat{H} = \hat{H}_{LGM} \; -\frac{2\,K}{N}\,\hat{S}_z^{\,2} \sum_{n=-\infty}^{\infty} \delta(t - n\tau)$$



- $[\hat{S}^2, \hat{H}] = 0, \;\; \vec{S} = \sum_i \vec{S}_i$ collective spin

- Floquet theory $\quad \hat{U} = \hat{U}_{\text{kick}} \exp\left[-i\hat{H}_{\text{LGM}}\,\tau\right]$

  $$\text{with } \hat{U}_{\text{kick}} \equiv \exp\left[-i\frac{2\,K}{N}\,\hat{S}_z^{\,2}\right]$$

- semiclassical limit $\hbar_{eff} \sim 1/N$

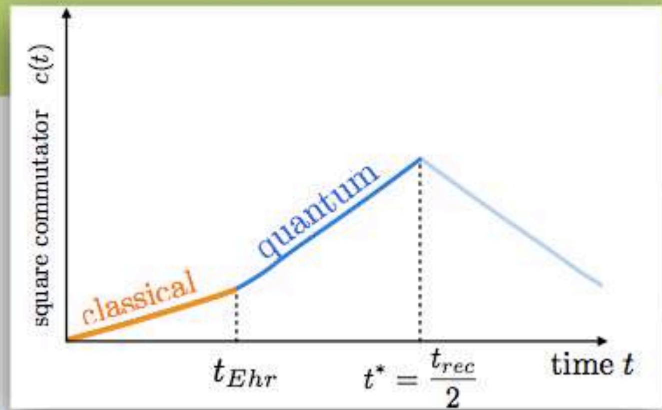
Haake - Springer, 2013

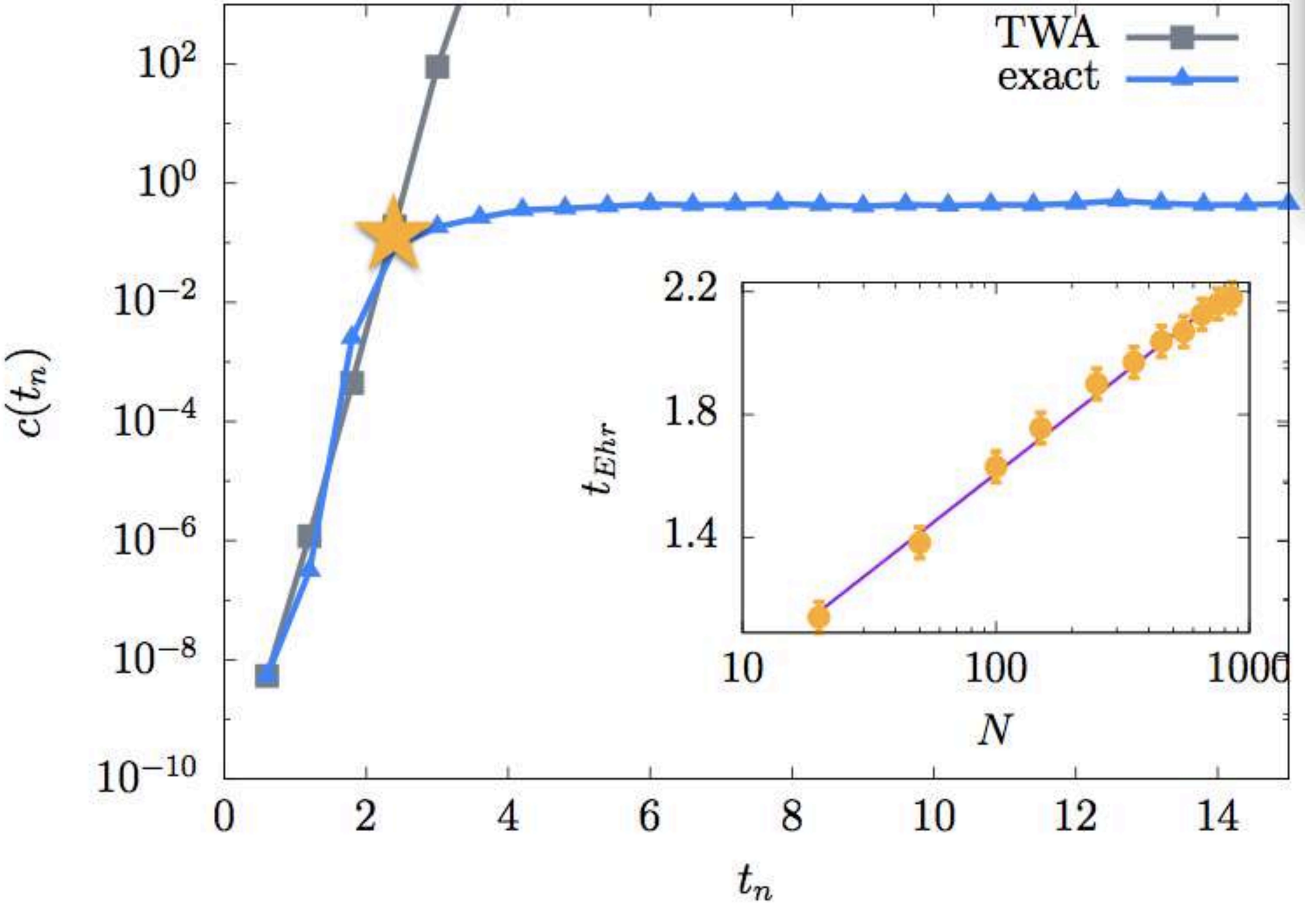
Scrambling in the chaotic top

$K = 20$

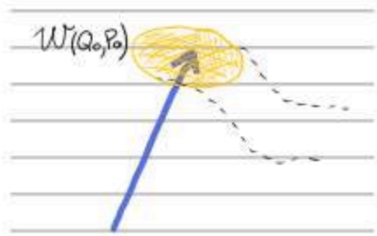
$$c(t) = -\langle [\hat{m}^z(t), \hat{m}^z]^2 \rangle$$

1. in chaotic systems
   quantum chaos = classical chaos
   for $t \leq t_{Ehr}$





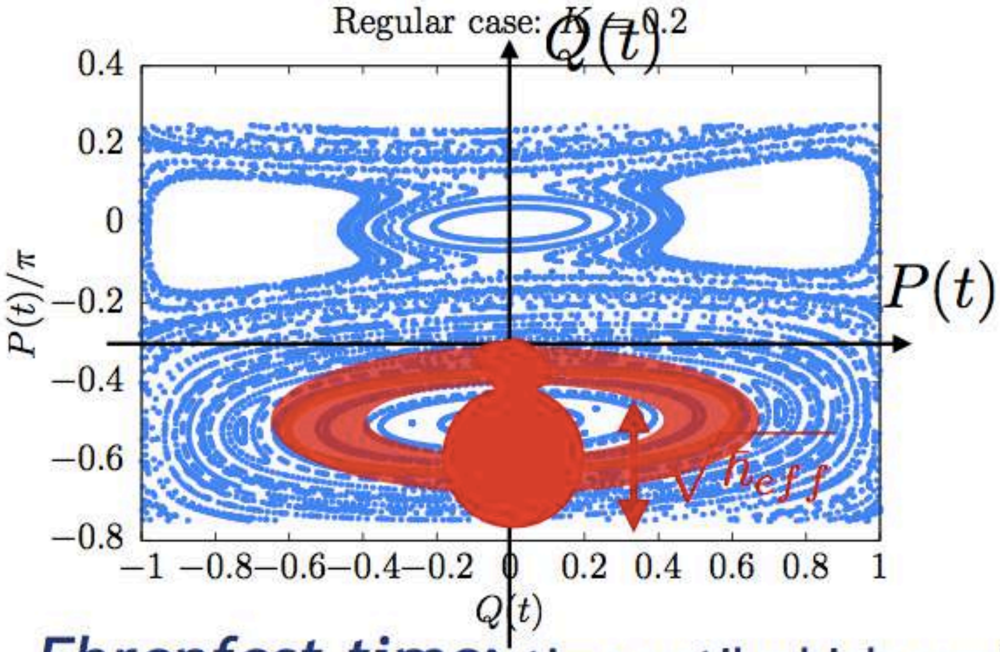
Truncated Wigner Approximation

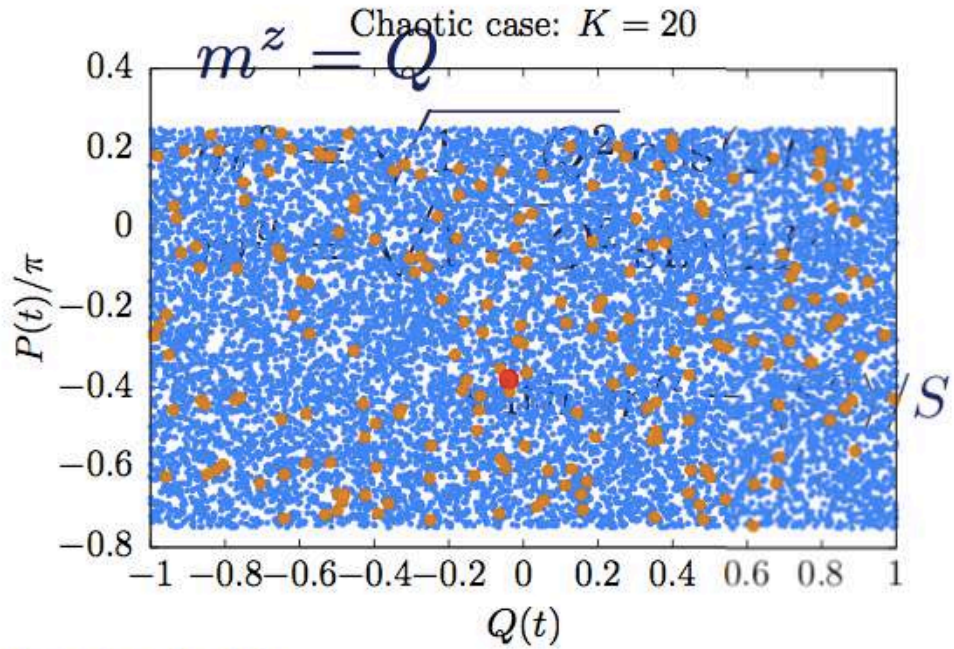


Classical and quantum chaos

$$\hat{U} = \exp\left[-i\frac{2K}{N}\hat{S}_z^2\right] e^{-i\hat{H}_{\mathrm{LGM}}\tau}$$

classical limit $\hbar_{eff} = \hbar/N$





Ehrenfest time: time until which semi-classics holds

$\sqrt{\hbar_{eff}}\, t \sim 1$ $\qquad t_{\mathrm{Ehr}} \sim \sqrt{N}$

$\sqrt{\hbar_{eff}} e^{\lambda t} \sim 1$ $\qquad t_{Ehr} \sim \log N$

Recurrence time: time at which the wave-packet regenerates

$t_{\mathrm{rec}} \sim N$ $\qquad t_{rec} \sim e^N$

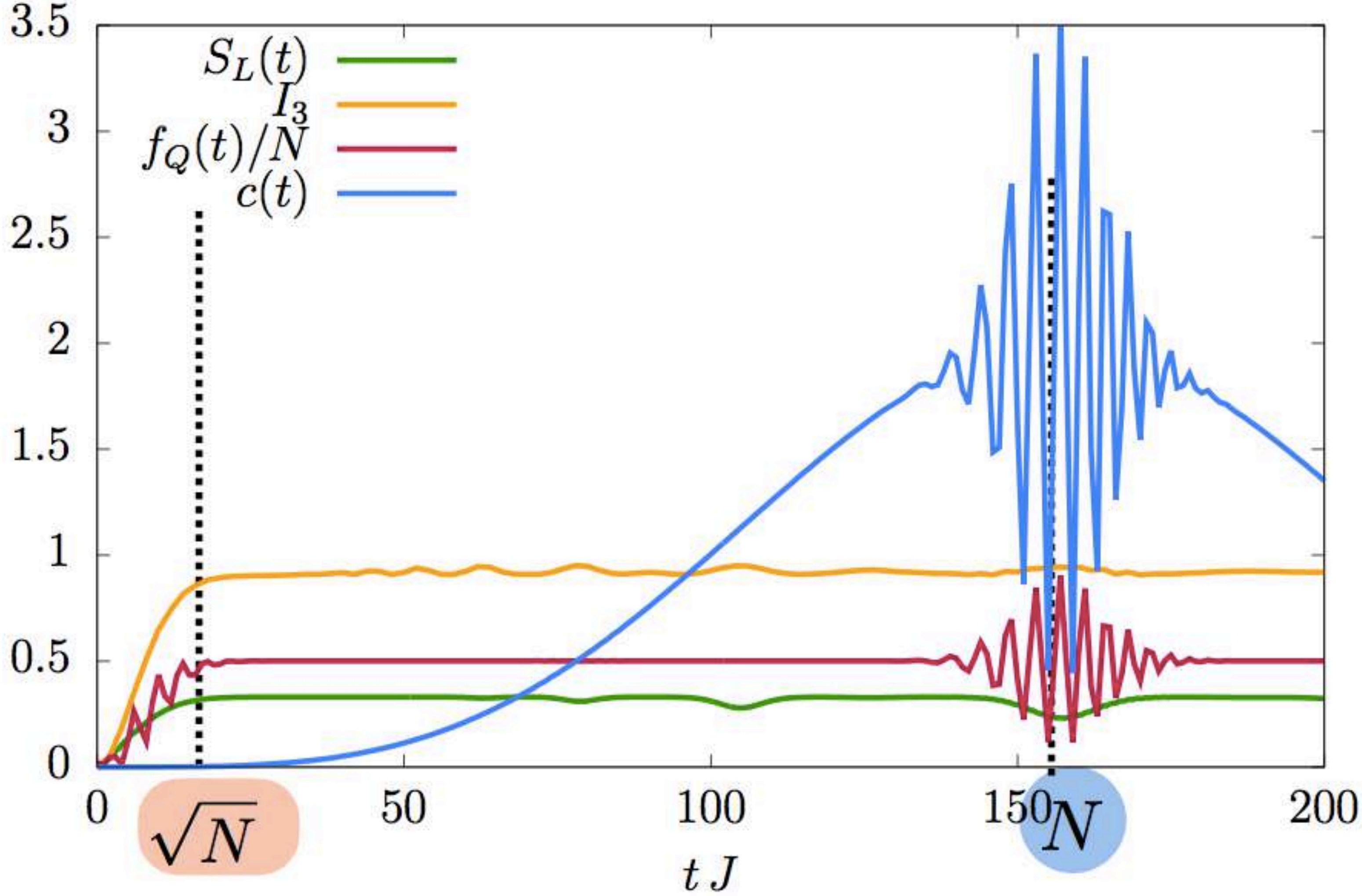
quantum spectral properties of Floquet spectrum: transition from Poisson to Wigner-Dyson distribution

Information dynamics in the LGM $K = 0$

$$c(t) = -\langle\, [\hat{m}^z(t), \hat{m}^z]^2 \,\rangle$$

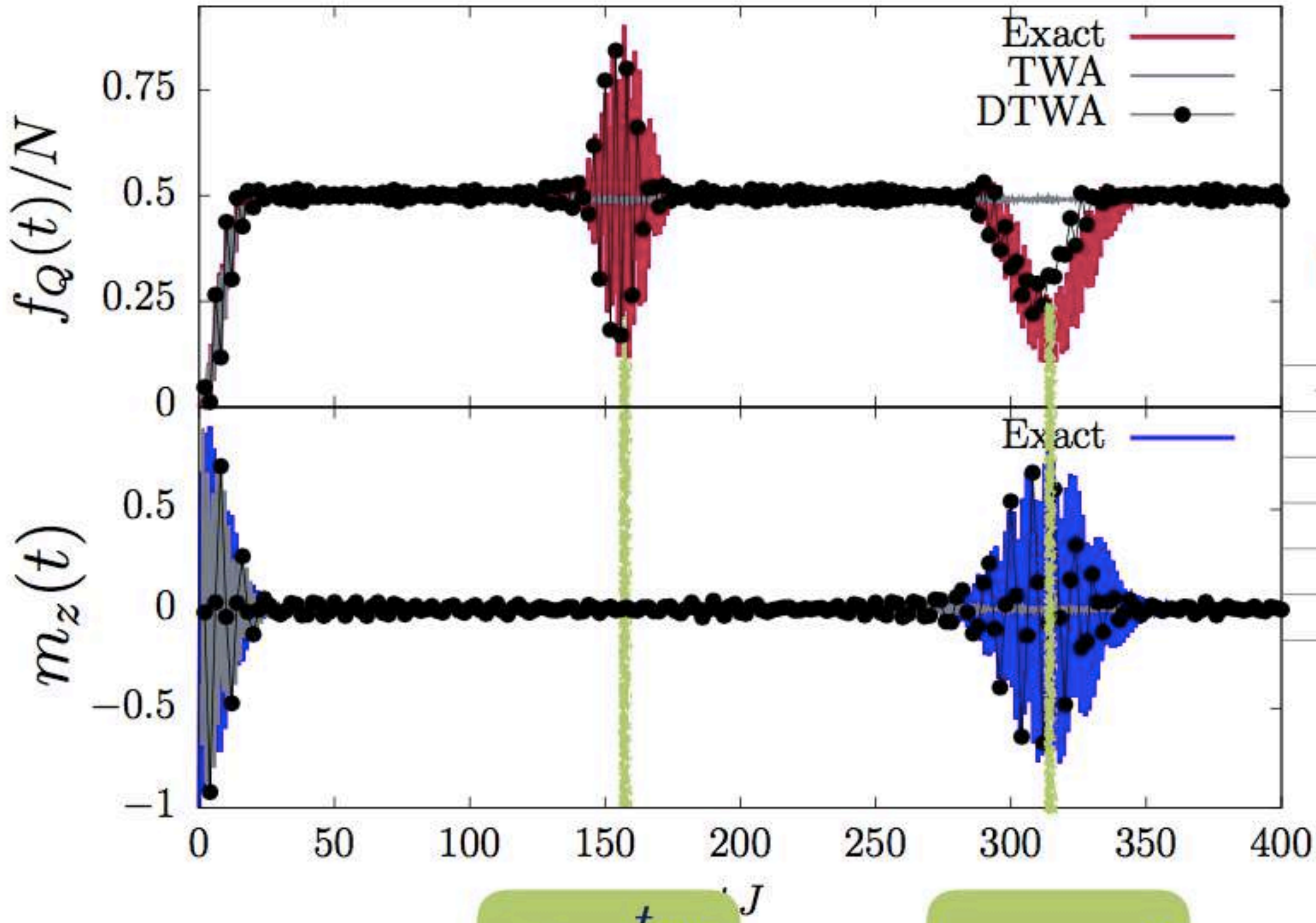
$$S_L = \mathrm{Tr}\,(\hat{\rho}_L \log \hat{\rho}_L\,)$$

2. scrambling goes beyond entanglement



Entanglement and semi-classics

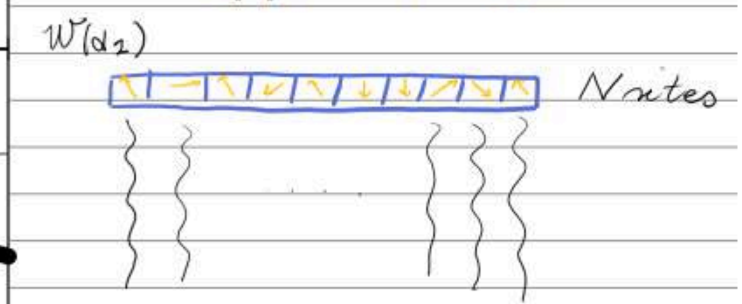
3.entanglement is a state dependent property



$$t^* = \frac{t_{rec}}{2}$$

$$t_{rec} = N$$

Discrete Truncated Wigner Approximation



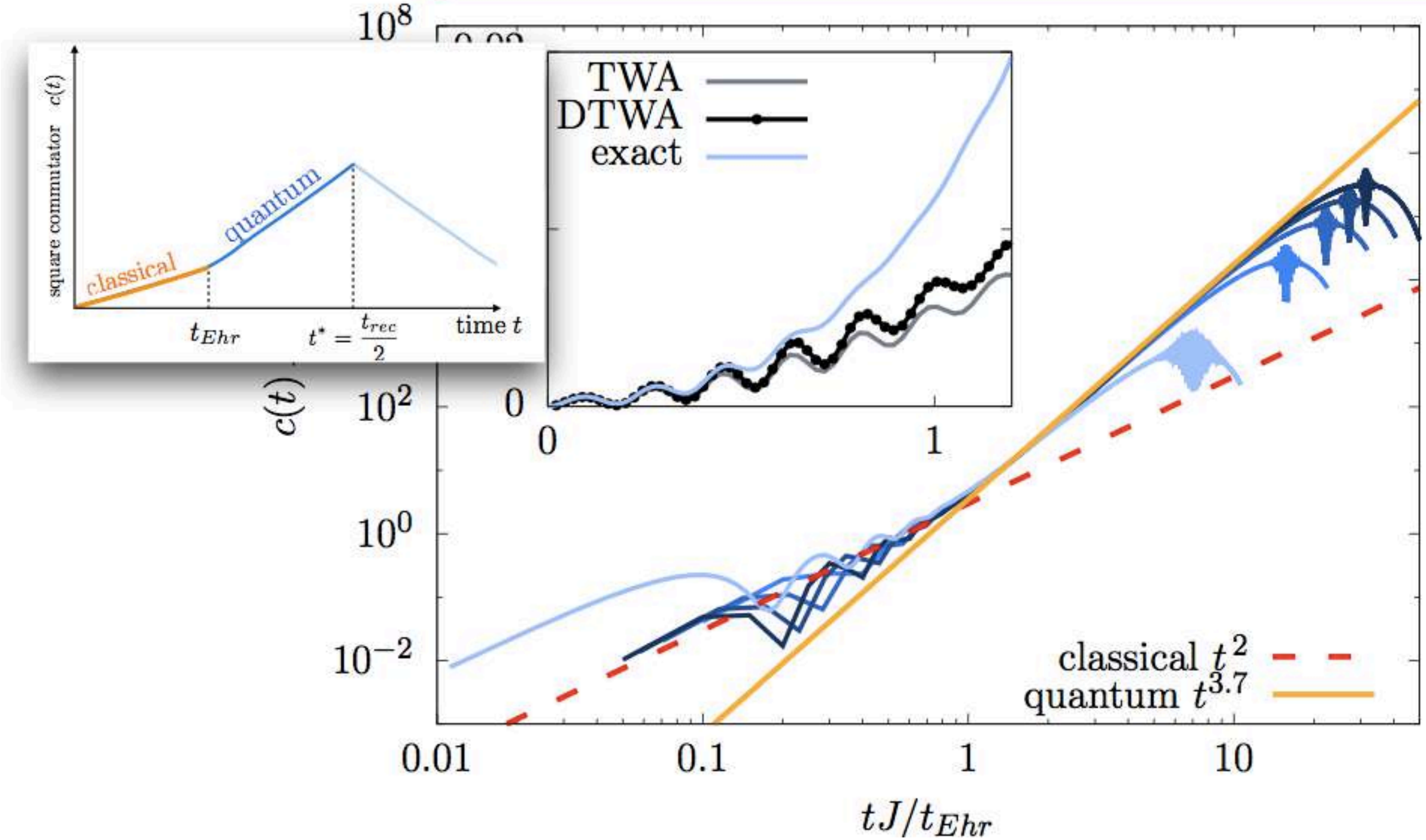
Scrambling beyond semiclassics

$c(t)$

$t \leq t_{Ehr}$ *semi-classical regime* $\sim \dfrac{t^2}{N^3}$

$t_{Ehr} \leq t \leq t^*$ *quantum regime* $c(t^*) = 2$

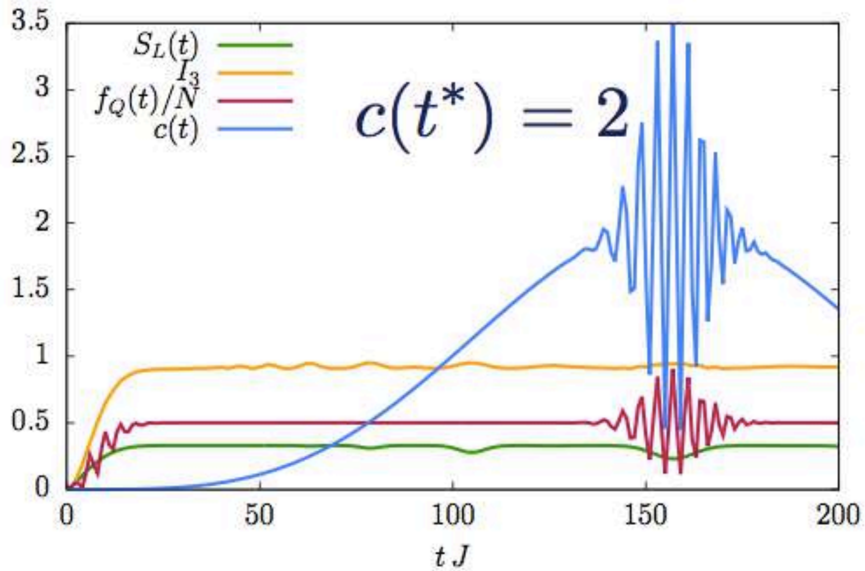
$t^* = \dfrac{t_{rec}}{2}$



Quantum regime and the operator's growth

$$c(t) = -\frac{1}{N^4} \langle \left[\hat{S}^z(t), \hat{S}^z\right]^2 \rangle$$

4. related to the operator's support growth: non-perturbative



$$\hat{S}_z(t) = \sum_{k=0}^{\infty} \frac{(-it)^k}{k!} [\hat{H}, [\ldots, [\hat{H}, \hat{S}_z]]]$$

string of length N

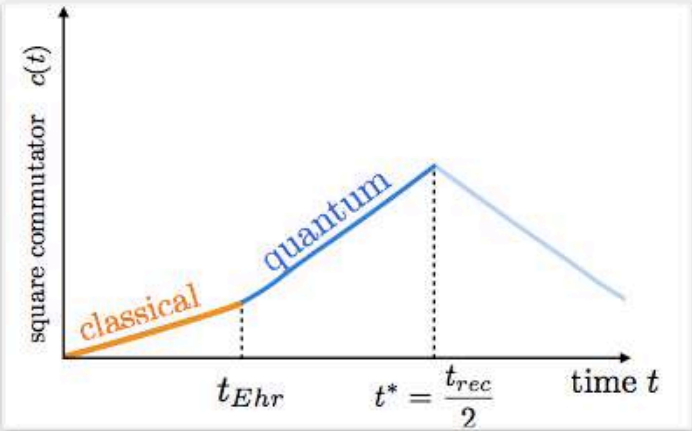
$$= \sum_{\alpha_1 \in \{x,y,z\}} a^{\alpha_1}(t)\,\widehat{S}^{\alpha_1} + \sum_{\alpha_1,\alpha_2} b^{\alpha_1\alpha_2}(t)\,\widehat{S}^{\alpha_1}\widehat{S}^{\alpha_2} + \cdots + \sum_{\alpha_1,\cdots,\alpha_N} z^{\alpha_1\cdots\alpha_N}(t)\,\widehat{S}^{\alpha_1}\widehat{S}^{\alpha_2}\cdots\widehat{S}^{\alpha_N}$$

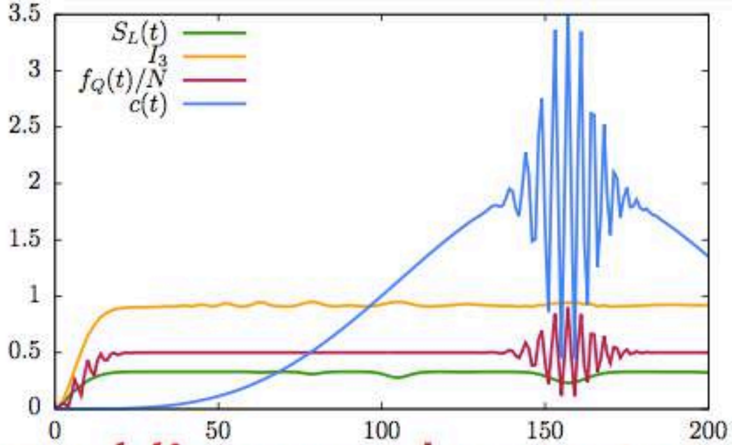
- scrambling always symmetric around

$$t^* = \frac{t_{rec}}{2}$$

Thanks!

Conclusions





scrambling: ***purely quantum, not entanglement***

1. in chaotic systems **quantum chaos = classical chaos** $t \leq t_{Ehe}$
2. scrambling goes beyond
3. entanglement is a state dependent property
4. related to the operator's support growth: non-perturbative

Perspectives

- breaking of integrability
- use operator space entropy

Wigner representation and TWA

Hilbert space ⟷ **Continous Phase Space**

- density matrix $\hat{\rho}$ — Wigner function $W(q,p)$
- operators $\hat{O}$ — Weyl transform $O_w(p,q)$

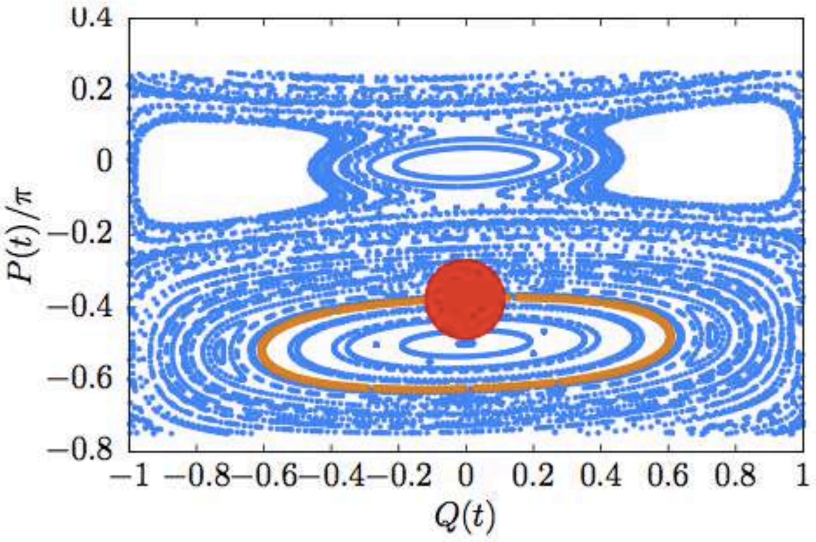
... ...

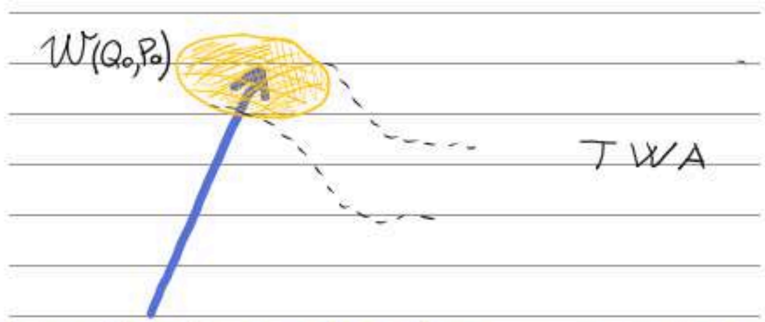
- expectation values

Truncated Wigner Approximation

$$\langle \hat{O}(t)\rangle = Tr[\,\hat{\rho}_0 \hat{O}(t)\,] \simeq \int dq_0\, dp_0\, W(q_0,p_0)\, O_w(q^{cl}(t), p^{cl}(t))$$

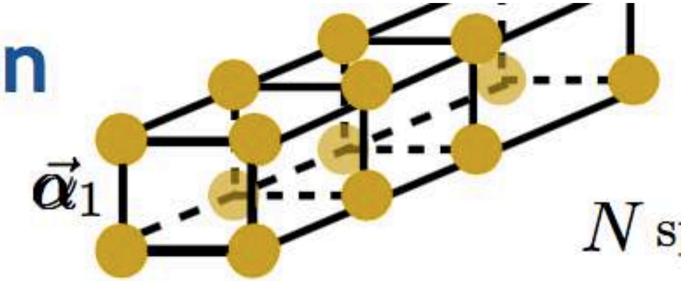
Montecarlo Sampling





classical evolution + average over the initial Wigner distribution

Discrete Wigner representation and DTWA



N spin $\frac{1}{2}$

Hilbert space ⟷ Discrete Phase Space

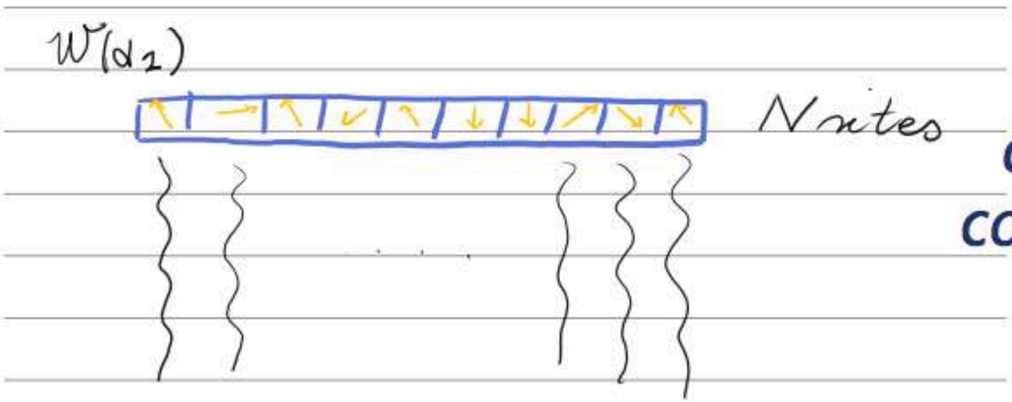
- density matrix $\hat{\rho}$ — Wigner function $W(\vec{\alpha})$
- operators $\hat{O}$ — Weyl transform $O_W(\vec{\alpha})$

... ...

- expectation values

$$\langle \hat{O}(t) \rangle = Tr[\,\hat{\rho}_0 \hat{O}(t)\,] = \sum_{\vec{\alpha}} W(\vec{\alpha}_0)\, O_W(\vec{\alpha}(t))$$

Montecarlo Sampling



discretization of the initial condition + classical evolution $3N$

Schachenmayer, Pikovski, Rey - Phys. Rev. X, 2015

Wootters - Annals of Physics, 198

Entanglement structure

Numerics with MPS-TDVP

matrix product state time-dependent variational principle

$$H(h) = -\frac{2J}{N(\alpha)} \sum_{i \neq j=1}^{N} \frac{S_i^z S_j^z}{|i-j|^\alpha} - 2h \sum_{i=1}^{N} S_i^x$$

$0 \leq \alpha < 1$	$1 < \alpha < 2$	$\alpha > 2$
$f_Q(\infty) \sim N$	$f_Q(t) \sim \text{const}$	$f_Q(t) \sim \text{const}$
$S_L(t) \sim \log t$	$S_L(t) \sim t^\beta$ with $\beta < 1$	$S_L(t) \sim t$
$S_L(\infty) \sim \log L$		$S_L(\infty) \sim L$
...	...	...

General structure!
induced by entanglement's
monogamy

Haegeman, Lubich, Oseledets, Vandereycken... - Phys. Rev. B, 2016

The Touchy Business of Formal Computations

Matteo Gallone

May 18, 2018

Junior Trieste Quantum Days 2018

Introduction

"No theorist in his right mind would have invented quantum mechanics unless forced by data"

– Craig Hogan

From Axioms:

- **Phase space:** Complex Hilbert space ($\mathcal{H} = L^2(\mathbb{R}^3, dx)$)
- **Observables:** Self-adjoint operators on $\mathcal{H}$
- **Time evolution:** Unitary 1-parameter group generated by Schrödinger equation

Unboundedness

Unboundedness is **unavoidable** in Quantum Mechanics:
Heisenberg's uncertainty principle:

$$[X, P] = \mathrm{i}$$

Proof:

$$[X^n, P] = \mathrm{i}nX^{n-1} \qquad \Rightarrow \qquad \|[X^n, P]\| = n\|X^{n-1}\|$$

$$\|[X^n, P]\| = \|X^n P - P X^n\| \leq 2\|X^n\|\|P\| \leq 2\|X^{n-1}\|\|X\|\|P\|$$

$$\frac{n}{2} \leq \|X\|\|P\| \qquad \forall n \in \mathbb{N}$$

Scientists in the '20-'30 need to develop the theory of **unbounded operators** (von Neumann, Stone, . . .)

Unbounded operators

Unbounded operator

An unbounded (= not necessarily bounded) operator is a linear map

$$T : \mathcal{D}(T) \subset \mathcal{H} \to \mathcal{H}$$

The assignment of the domain is **crucial**!

Different domains assigned to the same formal operator define **different operators:**

- eigenvalues
- scattering properties
- invertibility
- ...

What can go wrong? (1/2)

Statement

Time evolution associated to $\mathrm{i}\partial_t\psi = -\partial_x^2\psi$ is unitary (e.g. $\|\psi(t,x)\|_{L^2(\mathcal{I},dx)} = \|\psi(0,x)\|_{L^2(\mathcal{I},dx)}$) ($\mathcal{I} = (0,1) \subset \mathbb{R}$)

$$\begin{cases} \mathrm{i}\partial_t\psi(t,x) = -\partial_x^2\psi(t,x) \\ \psi(0,x) = e^{\frac{\mathrm{i}+1}{\sqrt{2}}x} \quad \in L^2(\mathcal{I},dx) \end{cases}$$

Look for solutions $\psi(t,x) = e^{\omega t}e^{kx}$:

Solution: $\psi(t,x) = e^{-t}e^{\frac{\mathrm{i}+1}{\sqrt{2}}x}$

$$\|\psi(t,x)\|^2_{L^2(\mathcal{I},dx)} = e^{-2t}\|\psi(0,x)\|^2_{L^2(\mathcal{I},dx)} \xrightarrow{t\to+\infty} 0$$

Source of problem: $\psi(0,x) \notin$ domain of self-adjointness!

What can go wrong? (2/2)

Statement

Eigenfunctions associated to different eigenvalues are orthogonal

$$-\mathrm{i}\frac{d}{dx}\psi_k(x) = k\psi_k(x) \qquad L^2(\mathcal{I}, dx)$$

If $k \in \mathbb{C}$, $\psi_k(x) = e^{\mathrm{i}kx} \in L^2(\mathcal{I}, dx)$ is an eigenfunction.
To see if they are orthogonal we need to evaluate

$$\langle \psi_k, \psi_j \rangle_{L^2(\mathcal{I},dx)} = \int_0^1 e^{-\mathrm{i}kx} e^{\mathrm{i}jx}\, dx$$

$$\langle \psi_j, \psi_k \rangle_{L^2(\mathcal{I},dx)} = \begin{cases} \frac{\mathrm{i}-\mathrm{i}e^{\mathrm{i}(j-k)}}{j-k} & j \neq k \\ 1 & j = k \end{cases}$$

Handbook of Definitions

Closed. T is closed iff $\mathcal{D}(T)$ with the *operatorial scalar product*:

$$\langle \psi, \varphi \rangle_T := \langle T\psi, T\varphi \rangle_{\mathcal{H}} + \langle \psi, \varphi \rangle_{\mathcal{H}}$$

is a Hilbert space (it is a Banach space).

Closable/Closure. $\overline{T}$, $\mathcal{D}(\overline{T}) = \overline{\mathcal{D}(T)}^{\|\cdot\|_T}$

Adjoint T^* If $\mathcal{D}(T)$ is dense in $\mathcal{H}$ then one defines

$$\mathcal{D}(T^*) := \{ f \in \mathcal{H} \,|\, \exists \eta \in \mathcal{H} \text{ s.t. } \langle f, T\varphi \rangle_{\mathcal{H}} = \langle \eta, \varphi \rangle_{\mathcal{H}}, \forall \varphi \in \mathcal{D}(T) \}$$

$$T^* f := \eta$$

Symmetric. $\langle \varphi, T\psi \rangle_{\mathcal{H}} = \langle T\varphi, \psi \rangle_{\mathcal{H}}$ $\forall \varphi, \psi \in \mathcal{D}(T)$. (equiv. $T \subset T^*$)

Self-adjoint. $T = T^*$ and $\mathcal{D}(T) = \mathcal{D}(T^*)$

Essentially self-adjoint. $\overline{T}$ is self-adjoint.

Self-adjoint extension. T symmetric, $T \subset \overline{T} \subset T_{\text{s.a.}} \subset T^*$.

Beyond toy examples

For differential and multiplicative operators, non-self-adjointness is due to

- boundary conditions
- singular points of the operator

In principle one can choose a lot of domains for unbounded operators. If we want to model nature there are some **natural** choices.

Natural domains

Formal operator $T = \sum_j c_j(\mathrm{i}\nabla)^j + V(x)$, Hilbert space $\mathcal{H} = L^2(\Omega)$, $\Omega \subset \mathbb{R}^n$ open:

- **Minimal domain:** $\mathcal{D}(T_{min}) = C_c^\infty(\Omega \setminus \Gamma)$.
  $\Gamma = \{x \in \Omega \mid V(x) \text{ is 'too singular'}\}$
- **Maximal domain:** $\mathcal{D}(T_{max}) = \{f \in \mathcal{H} | Tf \in \mathcal{H}\}$.
  T acts *distributionally*.

→ **Minimal operator** T_{min}**:** $(T, \mathcal{D}(T_{min}))$

→ **Maximal operator** T_{max}**:** $(T, \mathcal{D}(T_{max}))$

$$T_{min} \subset T_{max}$$

$$\begin{cases} T_{min} \text{ symmetric} \\ \mathcal{D}(T_{min}) \text{ is dense in } \mathcal{H} \end{cases} \implies T_{max} = T_{min}^*$$

Relativistic Quantum Mechanics

Dirac found the right equation to describe the motion of a $\frac{1}{2}$-spin particle in the relativistic regime:

$$i\hbar\partial_t\Psi(t,x) = H\Psi(t,x)$$

$$H_{free} = -ic\hbar\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta mc^2$$

$$\alpha_j = \begin{pmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}$$

$\Psi(t,x)$ is a **spinor**, i.e. $\Psi(t,x) \in L^2(\mathbb{R}^3,\mathbb{C}^4)$. This means

$$\Psi(t,x) = \begin{pmatrix} \Psi_1(t,x) \\ \Psi_2(t,x) \\ \Psi_3(t,x) \\ \Psi_4(t,x) \end{pmatrix} \qquad \begin{array}{l} e^- \text{ spin up} \\ e^- \text{ spin down} \\ ??? \\ ??? \end{array}$$

Relativistic Hydrogen Atom

Model of the hydrogen atom with relativistic kinetic energy

$$H_\nu = -\mathrm{i}\hbar c\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta mc^2 + \frac{\nu}{|x|}\mathbb{1}$$

It has been used to compute bond states energies:

$$E_n = mc^2\left(1 + \frac{\nu^2/c^2}{(n+\sqrt{1-(\nu^2/c^2)})^2}\right)^{-1/2}$$

- Correct non-relativistic limit

$$E_n - mc^2 \overset{c\to\infty}{\longrightarrow} -\frac{m\nu^2}{2(n+1)^2}$$

- Correct experimental prediction (fine-structure corrections)
- **Break-down of the formula:** If $|\nu| > c^2$ ($Z \approx 137$) we have **imaginary eigenvalues!**

History of the problem

$\underline{c = \hbar = 1}$

1948 - 1955: *Rellich* and *Kato* proved independently essentially self-adjointness for $|\nu| < \frac{1}{2}$

1970: *Rejtö* proved essentially self-adjointness for $|\nu| < \frac{3}{4}$

1971-1972: *Weidmann*, *Schmincke*, *Rejtö* and *Gustafsson* proved

- Essential-self adjointness for $|\nu| \leq \frac{\sqrt{3}}{2}$ (well-posedness)
- Non essential self-adjointness for $|\nu| > \frac{\sqrt{3}}{2}$ (ill-posedness)

2007: *Voronov*, *Gitman*, *Tyutin* classification 'a la von Neumann' of the extensions (abstract)

2013: *Hogreve* attempt of classification in terms of boundary conditions at $r = 0$

(M. Gallone, *Self-adjoint extensions of Dirac Operator with Coulomb Potential*, Advances in Quantum Mechanics, Springer, 2017)

Classification of extensions

2018: *M.G.* and *A. Michelangeli* proved that if $\nu \in (\frac{\sqrt{3}}{2}, 1)$ then

- $f \in \mathcal{D}(H^*)$ have asymptotics
  $f = \boldsymbol{a} r^{-\sqrt{1-\nu^2}} + \boldsymbol{b} r^{\sqrt{1-\nu^2}} + o(r^{1/2})$ as $r \to 0$
- The choice of $\gamma \in \mathbb{R} \cup \{\infty\}$ defines a self-adjoint realisation through the boundary condition

$$\boldsymbol{a} = (c_\nu \gamma + d_\nu)\boldsymbol{b}$$

  c_ν and d_ν are explicit (but not very illuminating!)
- Estimate of the ground state

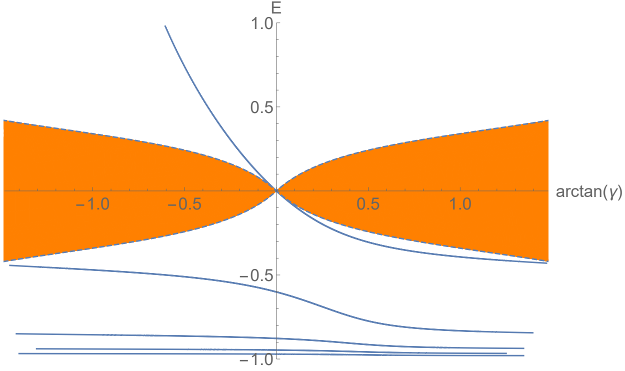
$$|E_0(\gamma)| = \frac{|\gamma|}{|\gamma|\sqrt{1-\nu^2}+1}$$

(M. Gallone and A. Michelangeli, *Self-adjoint realisations of the Dirac-Coulomb Hamiltonian for heavy nuclei*, Analysis and Math Phys, 2018)

Eigenvalues

Boundary condition $\Longrightarrow$ explicit formula for EV

$$\mathfrak{F}(E) = c_\nu \gamma + d_\nu$$



(M. Gallone and A. Michelangeli, *Discrete spectra for critical Dirac-Coulomb Hamiltonians*, Journal Math Phys, 2018)

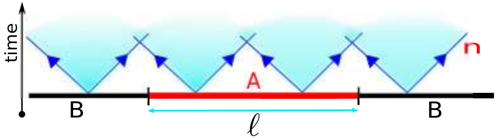
Thank you for your attention

Entanglement and thermodynamics in out-of-equilibrium systems

Vincenzo Alba[1]

[1]SISSA, Trieste

Trieste, 18/05/2018



Outline

[V.A. and P. Calabrese, PNAS **114**, 7947 (2017)]

- **Complexity** of out-of-equilibrium quantum matter.
- **Entanglement** and **quenches**.
- **Goal: Entanglement dynamics after quantum quenches.**
- **Semiclassical** picture & **Integrability**.
- **von Neumann** vs **Rényi** entropies.

[V. Alba and P. Calabrese, Phys. Rev. B 96, 11541 (2017)]

[V. Alba and P. Calabrese, J. Stat. Mech. (2017) 113105]

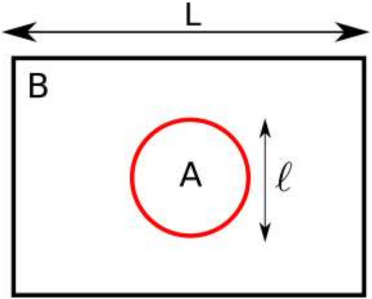
[V. Alba and P. Calabrese, arXiv:1712.07529]

[V. Alba, arXiv:1706.00020]

SISSA

MARIE CURIE ACTIONS

- **Question**: How do simple descriptions (**thermodynamics**) emerge in out-of-equilibrium **isolated** sytems?



$A \cup B =$ **isolated** universe

Unitary dynamics under Hamiltonian H

$L, \ell \to \infty, \mathrm{with}\, \ell \ll L$

$time \to \infty$

- Long-time limit of local reduced density matrix? Is it **thermal?**

$$\rho_A \equiv \mathrm{Tr}_B \rho_{A\cup B}$$

Wonders of out-of-equilibrium systems

P. P. Rubens, *Vulcan forging the Thunderbolts of Jupiter* (1637), Prado Museum

"sudden" global manipulation

Quantum quench, Floquet dynamics, adiabatic quench (ramping)

isolated quantum system (T=0)

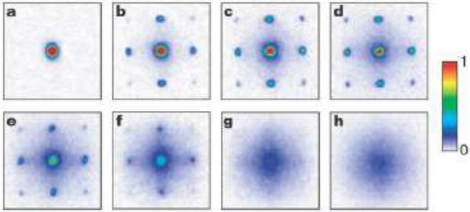
Theory toolbox:

- Integrability
- DMRG
- Exact diagonalization
- Field Theory methods

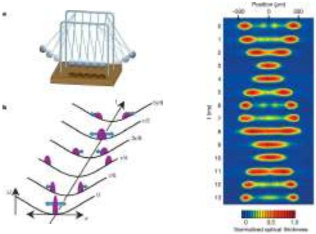
Challenge: No **unifying** theory framework

Out of equilibrium physics in cold-atom experiments

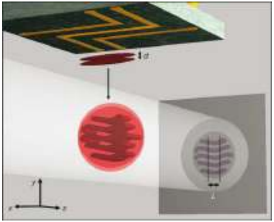
[Greiner, Nature (2002)]



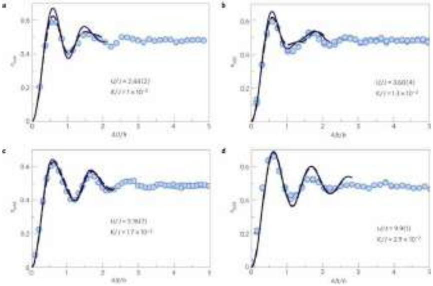
[Kinoshita et al., Nature 440, 900 (2006)]



[Hofferberth, Nature (2007)]



[Trotzky, Nature Phys. (2012)]



Quantum quench protocol

- Initial state $|\boldsymbol{\Psi_0}\rangle \Rightarrow$ **unitary** evolution under a many-body Hamiltonian $\boldsymbol{\mathcal{H}}$

$\{|\psi_\alpha\rangle\}$ eigenstates of $\mathcal{H}$

$$|\Psi_0\rangle = \sum_\alpha c_\alpha |\psi_\alpha\rangle \qquad |\Psi(t)\rangle = \sum_\alpha e^{iE_\alpha t} c_\alpha |\psi_\alpha\rangle$$

$$c_\alpha \equiv \langle \Psi_0 | \psi_\alpha \rangle$$

- For a generic observable $\widehat{\mathcal{O}}$:

$$\langle \Psi(t) | \widehat{\mathcal{O}} | \Psi(t) \rangle = \sum_{\alpha,\beta} e^{i(E_\alpha - E_\beta)t} c^*_\alpha c_\beta \widehat{\mathcal{O}}_{\alpha\beta}$$

- **Long time** $\Rightarrow$ **diagonal ensenble**.

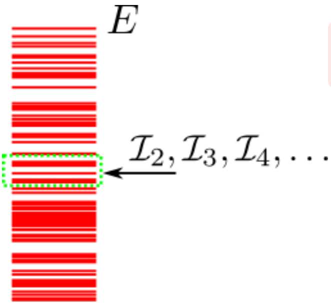
$$\boxed{\overline{\langle \Psi(t) | \widehat{\mathcal{O}} | \Psi(t) \rangle} = \langle \hat{\mathcal{O}} \rangle_{DE} = \sum_\alpha |\langle \Psi_0 | \psi_\alpha \rangle|^2 \widehat{\mathcal{O}}_{\alpha\alpha}}$$

Equilibration in integrable models

- **Integrability** $\Rightarrow$ **Local** (quasi-local) **conserved** quantities $\mathcal{I}_j$.

$$[\mathcal{H}, \mathcal{I}_j] = 0, \ \forall j \quad \text{and} \quad [\mathcal{I}_j, \mathcal{I}_k] = 0, \ \forall j, k \qquad \mathcal{I}_2 \equiv \mathcal{H}$$

- Include extra charges in Gibbs $\Rightarrow$ **Generalized Gibbs Ensemble** (GGE).



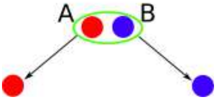
$$\rho^{GGE} = \frac{1}{Z} \exp\left(\sum_j \beta_j \mathcal{I}_j \right)$$

[Jaynes, 1957;Rigol,2008]

- **Generalized microcanonical** principle.

- Einstein-Podolsky-Rosen paradox:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle) + |\downarrow\uparrow\rangle$$



- Perfect anticorrelated spin measurements.

RESEARCH ARTICLE

QUANTUM OPTICS

Satellite-based entanglement distribution over 1200 kilometers

Juan Yin, Yuan Cao, Yu-Huai Li, Sheng-Kai Liao, Liang Zhang, Ji-Gang Ren, Wen-Qi Cai, Wei-Yue Liu, Bo Li, Hui Dai, Guang-Bing Li, Qi-Ming Lu, Yun-Hong Gong, Yu Xu, Shuang-Lin Li, Feng-Zhi Li, Ya-Yun Yin, Zi-Qing Jiang, Ming Li, Jian-Jun Jia, Ge Ren, Dong He, Yi-Lin Zhou, Xiao-Xiang Zhang, Na Wang, Xiang Chang, Zhen-Cai Zhu, Nai-Le Liu, Yu-Ao Chen, Chao-Yang Lu, Rong Shu, Cheng-Zhi Peng, Jian-Yu Wang, Jian-Wei Pan

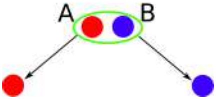
Long-distance entanglement distribution is essential for both foundational tests of quantum physics and scalable quantum networks. Owing to channel loss, however, the previously achieved distance was limited to ~100 kilometers. Here we demonstrate satellite-based distribution of entangled photon pairs to two locations separated by 1203 kilometers on Earth, through two satellite-to-ground downlinks with a summed length varying from 1600 to 2400 kilometers. We observed a survival of two-photon entanglement and a violation of Bell inequality by 2.37 ± 0.09 under strict Einstein locality conditions. The obtained effective link efficiency is orders of magnitude higher than that of the direct bidirectional transmission of the two photons through telecommunication fibers.

Science 356, 1140 (2017)

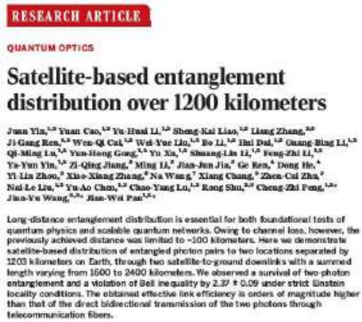
Entanglement: quantum mechanics at its strangest

- Einstein-Podolsky-Rosen paradox:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle) + |\downarrow\uparrow\rangle)$$



- Perfect anticorrelated spin measurements.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Science 356, 1140 (2017)

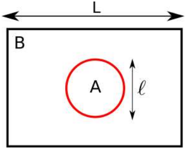
- Haiku view on entanglement:

Up here down there, these bonds are stronger than time. N.B.

Entanglement entropy in many-body systems

- Consider a quantum system in d dimensions in a **pure** state $|\Psi\rangle$

$$\rho \equiv |\Psi\rangle\langle\Psi|$$



- If the system is **bipartite**:

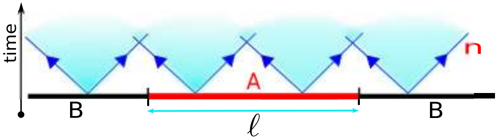
$$H = H_A \otimes H_B \to \rho_A = Tr_B \rho$$

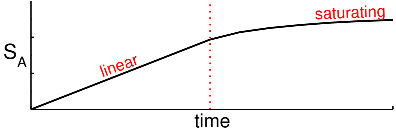
- How to quantify the entanglement (**quantum** correlations) between A and B?
  - **von Neumann** entropy $S_A = -Tr\rho_A \log \rho_A = -\sum_i \lambda_i \log \lambda_i$
  - **Rényi** entropies $S_A^{(n)} = -\frac{1}{n-1}\log(Tr\rho_A^n) = -\frac{1}{n-1}\log(\sum_i \lambda_i^n)$

Entanglement dynamics: Semiclassical picture

- **Extensive** amount of energy $\Rightarrow$ **quasi-particles** produced uniformly in the initial state.

[Calabrese, Cardy, 2005]





$$S_A(t) \propto 2t \int_{2|v|t<\ell} d\lambda v(\lambda) f(\lambda) + \ell \int_{2|v|t>\ell} d\lambda f(\lambda)$$

- Requires quasi-particles **group velocities** $v(\lambda)$
- $\boldsymbol{f(\lambda)}$ **cross-section** for quasi-particle production.
- Exact for free models.

[Fagotti, Calabrese, 2008]

Integrable models (*à la* Bethe ansatz)

- **Integrability** $\Rightarrow$ **stable** families of "**single particle**" excitations.

$$\boldsymbol{\lambda_{n,j}} = \text{particle quasimomentum} \approx \textbf{rapidity}.$$

- Generic eigenstate:

$$|\{\lambda_{n,j}\}\rangle$$

- Thermodynamic limit $\Rightarrow$ **macrostate** $\Rightarrow$ **particle** and **hole** densities

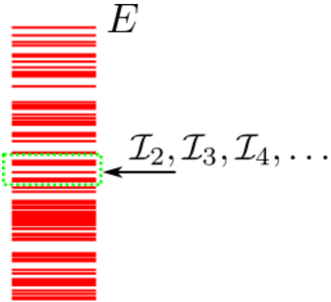
$$|\{\rho_n(\lambda), \rho_n^{(h)}(\lambda)\}\rangle$$

- $\#$ equivalent microscopic eigenstates $\Rightarrow$ Yang-Yang **entropy**

$$S_{YY} \equiv L \sum_n \int d\lambda [\rho_n^{(t)} \log \rho_n^{(t)} - \rho_n \log \rho_n - \rho_n^{(h)} \log \rho_n^{(h)}]$$

Quenches in integrable models

- Key idea: **Steady state** $\Rightarrow$ **macrostate $|\rho_n\rangle$**.



$e^{S[\rho_n]} = \#$ **of representative eigenstates**

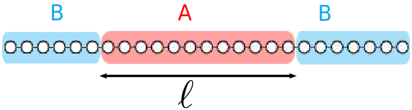
$S[\rho_n] =$ **thermodynamic entropy**

$|\rho_n\rangle$

- **Integrability** $\Rightarrow$ $|\rho_n\rangle$ and $S[\rho_n]$ can be determined **analytically**.

Steady state entanglement entropy

- Steady-state **entanglement** entropy density is the **thermodynamic** entropy.



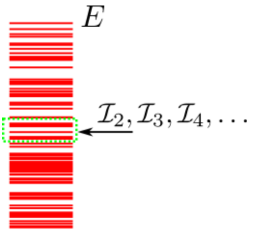
$$\boxed{S_A/\ell = (\mathrm{Tr}\rho^{GGE}\log\rho^{GGE})/L = \sum_n \int d\lambda s_n(\lambda)}$$

- Cross section for quasi-particle production is fixed $\boldsymbol{f(\lambda) = s_n(\lambda)}$:

$$\boxed{S_A(t) \xrightarrow{t\to\infty} \ell \sum_n \int d\lambda s_n(\lambda)}$$

Entangling quasi-particles

- How to identify the **entangling** quasi-particles?

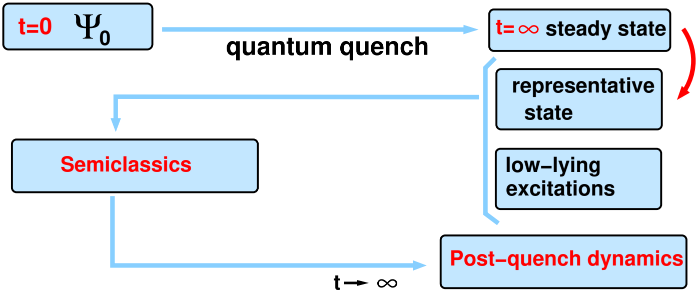


$|\rho_n\rangle$

[J.-S. Caux and F. Essler, Phys. Rev. Lett. 110, 257203 (2013)]

- **Local** observables $\Rightarrow$ dynamics determined by **low-lying** excitations around steady state $|\rho_n\rangle$.

Theoretical program



$$S_A(t) \propto \sum_k \left[t \int_{|v_k|t<\ell} d\lambda v_k(\lambda) s_k(\lambda) + \ell \int_{|v_k|t>\ell} d\lambda s_k(\lambda) \right]$$

- Spin-1/2 anisotropic Heisenberg (XXZ) chain.

$$\mathcal{H}_{XXZ}=\sum_{i=1}^{L}(S_i^+S_{i+1}^-+S_i^-S_{i+1}^++\Delta S_i^zS_{i+1}^z)$$

$\Delta\geq 1$

- Initial states:

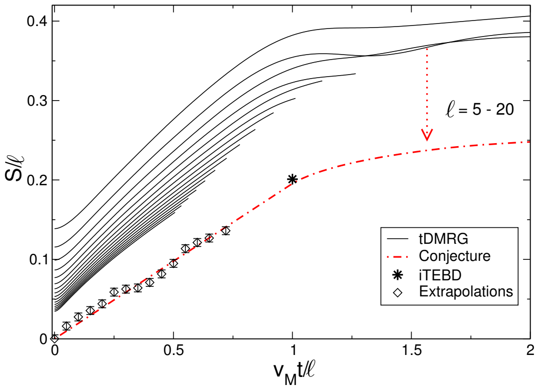
Tilted ferromagnet $|UP,\vartheta\rangle\equiv\frac{1}{\sqrt{2}}e^{i\vartheta/2\sum_j\sigma_j^y}|\uparrow\uparrow\cdots\rangle$

Tilted Néel $|N,\vartheta\rangle\equiv\frac{1}{\sqrt{2}}e^{i\vartheta/2\sum_j\sigma_j^y}(|\uparrow\downarrow\rangle^{\otimes L/2}+|\downarrow\uparrow\rangle^{\otimes L/2})$

Majumdar-Ghosh (Dimer) $|MG\rangle\equiv(\frac{|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle}{\sqrt{2}})^{\otimes L/2}$

Numerical checks: Full time evolution

- XXZ chain with $\Delta = 2$: Quench from Néel state.



- Fairly good agreement apart from finite size (time) corrections.

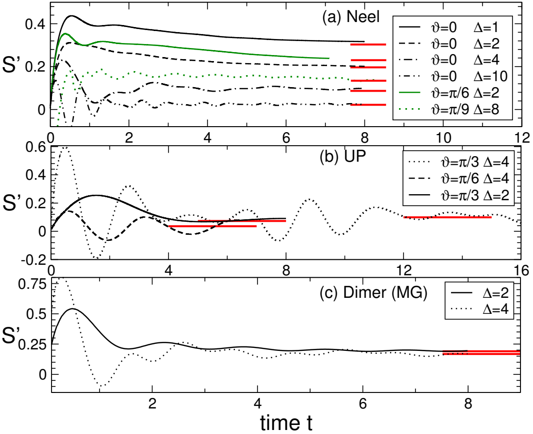
$$S_A(t) \propto \sum_k \Big[t \int_{|v_k|t<\ell} d\lambda v_k(\lambda) s_k(\lambda) + \ell \int_{|v_k|t>\ell} d\lambda s_k(\lambda) \Big]$$

Numerical checks: Linear growth

- Quench in the *XXZ* chain.

iTEBD from [Fagotti et al., 2015]

$$S' \equiv \frac{dS(v_M t)}{d(v_M t)}$$



$$S_A(t) \propto \sum_k \Big[\boxed{t \int_{|v_k|t<\ell} d\lambda\, v_k(\lambda) s_k(\lambda)} + \ell \int_{|v_k|t>\ell} d\lambda\, s_k(\lambda) \Big]$$

Conclusions

- **Entanglement** dynamics after quantum **quenches** in **integrable** models.
- Improved **Semiclassical** picture using **integrability**.
- Entanglement dynamics encoded in the **steady state** and **low-lying** excitations around it.

Thanks!

Non-linear Schrödinger equation with point interactions

Raffaele Scandone (SISSA Trieste)

Trieste Junior Quantum Days, 18 May 2018

Based on joint works with Vladimir Georgiev, Alessandro Michelangeli and Alessandro Olgiati.

Heuristic model

Formally, consider the equation

$$i\partial_t u = -\Delta_x + \sum_{j=1}^{N} \mu_j \, \delta(x - y_j) + \mathcal{N}(u),$$

where $\{y_1, \ldots y_N\}$ are distinct points in $\mathbb{R}^d$, which supports delta-like interactions of strenght $\{\mu_1, \ldots \mu_N\}$, and $\mathcal{N}(u)$ is a non-linear interaction potential.

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Why we consider point interactions?

Why we consider non-linear potentials?

How to give a rigorous meaning to the equation?

Point interactions, motivations

$$-\Delta_x + \sum_{j=1}^{N} \mu_j \, \delta(x - y_j)$$

provides an heuristic model for a quantum particle subject to a "contact potential", created by point sources of strenght μ_y centered at y_j.

- Kronig and Penney (1931) consider the 1D case as a model for a non-relativistic electron moving in a fixed crystal lattice.
- Bethe, Peierls (1935) and Thomas (1935) consider the 3D case with $y = 0$. Introducing the center of mass and relative coordinates, this provides a model for a deuteron with idealized zero-range nuclear force between the nucleons.
- Appears in many contexts: nuclear physics, solid state physics etc.
- Provide "solvable" approximation of more complicated and realistic phenomena, governed by very short range interactions

Non-linear potentials, motivations

Consider the 3D *many-body* Hamiltonian

$$H_N := \sum_{j=1}^{N} -\Delta_{x_j} + \sum_{1\leq j<k\leq N} w_N(x_j - x_k),$$

where w_N is a pontential governing the interaction between the particles. Assume *Bose-Einstein condensation* at time $t = 0$:

$$\psi_0^{(N)}(x_1, \ldots, x_N) \approx \prod_{j=1}^{N} u_0(x_j), \quad N \gg 1.$$

Then we have condensation at any time $t > 0$:

$$\left(e^{itH_N}\psi_0^{(N)}\right)(x_1, \ldots, x_N) \approx \prod_{j=1}^{N} u(t, x_j), \quad N \gg 1,$$

$$\begin{cases} i\partial_t u = -\Delta u + \mathcal{N}(u) \\ u(0, x) = u_0(x), \end{cases} \qquad \mathcal{N}(u) = \begin{cases} (w * |u|^2)u & w_N = N^{-1}w \\ |u|^2 u & w_N = N^2 w(Nx) \end{cases}$$

Non-linear potential with point interactions

$$H_{N,\varepsilon} := \sum_{j=1}^{N} \big(-\Delta_{x_j} + V_\varepsilon(x_j)\big) + \sum_{1 \le j < k \le N} w_N(x_j - x_k),$$

where V_ε are smooth potentials, shrinking around the origin in such a way to create a delta-like profile as $\varepsilon \to 0$. Assume Bose-Einstein condensation at time $t = 0$.

- **Is condensation preserved, at least for short times?**
- **Can we rigorous prove that the evolution of the condensate is governed by the equation**

$$i\partial_t u = -\Delta u + \mu\delta(x) + \mathcal{N}(u)$$

in the limit $N \to +\infty$ **and** $\varepsilon \to 0$?

Work in progress with A. Michelangeli and A. Olgiati. As a first step, we need to show existence of solutions for the limit equation.

Rigorous construction of point interactions

Assume, for simplicity, a single center at the origin.

- In dimension one, consider the quadratic form

$$Q(f,g) := \int_{\mathbb{R}} \overline{\partial_x f} \cdot \partial_x g \, dx + \mu \overline{f(0)} g(0), \quad f,g \in H^1(\mathbb{R}).$$

  From Q, we realise $-\Delta_x + \mu\delta(x)$ as a *self-adjoint operator* on $L^2(\mathbb{R})$.

- In higher dimension, we need a different approach. Suppose $d = 3$. The *symmetric* operator $-\Delta|_{C_0^\infty(\mathbb{R}^3 \setminus \{0\})}$ has a one-parameter family of *self-adjoint extension* $\{-\Delta_\alpha\}_{\alpha \in (-\infty, +\infty]}$. For $\lambda > 0$,

$$\mathcal{D}(-\Delta_\alpha) = \left\{ \psi \in L^2(\mathbb{R}^3) \, : \, \psi = F_\lambda + \frac{F_\lambda(0)}{\alpha + \frac{\sqrt{\lambda}}{4\pi}} \frac{e^{-\sqrt{\lambda}|x|}}{4\pi|x|} \, : \, F_\lambda \in H^2(\mathbb{R}^3) \right\}$$

$$(-\Delta_\alpha + \lambda)\psi = (-\Delta + \lambda) F_\lambda$$

Asymptotics of the wave function

- The parameter α is related to the *scattering lenght* of the system at the centre of interaction. Indeed, a generic element $\psi \in \mathcal{D}(-\Delta_\alpha)$ satisfies the so called *Bethe-Peierls contact condition*

$$\psi(x) \underset{x \to 0}{\sim} \frac{1}{|x|} - \frac{1}{s}, \quad s = -(4\pi\alpha)^{-1},$$

  which is typical for the low-energy behaviour of an eigenstate of the Schrödinger equation for a quantum particle subject to a very short (virtually zero) range potentials, centered at the origin and with s-wave scattering lenght s.
- When $\alpha = +\infty$, then no actual interaction is present at the origin (the s-wave scattering lenght is zero), and we recover the free Laplacian in $L^2(\mathbb{R}^3)$.

Approximation with regular operators

Let V smooth and compactly supported, and assume that $-\Delta + V$ has a *zero energy resonance*, namely a funtion $\psi \in L^1 \setminus L^2$ such that

$$(-\Delta + V)\psi = 0.$$

Existence of a zero energy resonance is related to *confining* property of the potential V. In particular, V must have a negative part.
Define, for $\varepsilon > 0$ and a function $\lambda : \mathbb{R} \to \mathbb{R}$, with $\lambda(0) = 1$,

$$V_\varepsilon := \frac{\lambda(\varepsilon)}{\varepsilon^2} V\Big(\frac{x}{\varepsilon}\Big)$$

The potential is shrinking around the origin. N.B. the scaling is **not** that of a delta function, but is *weaker*. We have, in a suitable resolvent sense

$$\lim_{\varepsilon \to 0} (-\Delta + V_\varepsilon) = -\Delta_\alpha$$

Rigorous formulation of the equation

Consider the Cauchy problem

$$\begin{cases} i\partial_t u = -\Delta_\alpha u + \mathcal{N}(u), & t \in \mathbb{R}, x \in \mathbb{R}^3, \\ u(0,\cdot) = f \in X \end{cases}$$

where X is a suitable Hilbert space, for example $L^2(\mathbb{R}^3)$.

- Since $-\Delta_\alpha$ is self-adjoint, we have a unitary evolution $e^{-it\Delta_\alpha}f$.
- Integral formulation of the equation:

$$u(t,x) = e^{it\Delta_\alpha}f - i\int_0^t e^{i(t-s)\Delta_\alpha}\mathcal{N}(u)(s)ds$$

- We search for solution $u \in \mathcal{C}(I, X)$ of the integral equation, for some time interval I.

Energy space

Define, for $\alpha > 0$, the Banach space

$$H^1_\alpha := \mathcal{D}((-\Delta_\alpha)^{1/2}) \quad \|\psi\|_{H^1_\alpha} := \|(-\Delta_\alpha + 1)^{1/2}\psi\|_{L^2(\mathbb{R}^3)}$$

- When $\alpha = +\infty$, we recover the Sobolev space $H^1(\mathbb{R}^3)$.
- We have an explicit characterisation ([Georgiev, Michelangeli, S.] for a discussion of the general fractional case)

$$H^1_\alpha = \left\{ \psi \in L^2(\mathbb{R}^3) \,:\, \psi = F_\lambda + c\frac{e^{-\sqrt{\lambda}|x|}}{4\pi|x|} \,:\, F_\lambda \in H^1(\mathbb{R}^3), c \in \mathbb{C} \right\}$$

$$\left\| F_\lambda + c\frac{e^{-\sqrt{\lambda}|x|}}{4\pi|x|} \right\|^2_{H^1_\alpha} \sim \|F\|^2_{H^1} + (1+\alpha)|c|^2$$

- The $\|\cdot\|_{H^1_\alpha}$-norm is preserved by the unitary evolution $e^{-it\Delta_\alpha}$.
- We will use H^1_α as the energy space also for the non-linear problem.

Existence of solutions

Theorem (Michelangeli, Olgiati, S., 2018)

Let $w = |x|^{-\gamma}$, with $0 < \gamma < \frac{1}{2}$. Then the Cauchy problem

$$\begin{cases} i\partial_t u = -\Delta_\alpha u + (w * |u|^2)u, & t \in \mathbb{R}, x \in \mathbb{R}^3, \\ u(0,\cdot) = f \in H^1_\alpha \end{cases}$$

has a unique solution $u \in \mathcal{C}([0, T^*), H^1_\alpha)$, defined on a maximal time interval $[0, T^*)$.

We have the *blow-up* alternative:

$$T^* < +\infty \iff \lim_{t \uparrow T^*} \|u(t)\|_{H^1_\alpha} = +\infty$$

As long as $\|u(t)\|_{H^1_\alpha}$ stay bounded, the solution can be extended in time.

Global solutions

Defone the *mass* and the *energy*:

$$\mathcal{M}(u) := \int_{\mathbb{R}^3} |u|^2 dx$$
$$\mathcal{E}(u) :=< -\Delta_\alpha u, u > +\frac{1}{2}\int_{\mathbb{R}^3} (w * |u|^2)|u|^2 dx$$

- Formally, if u is a solution of the Cauchy problem, then $\mathcal{M}(u(t))$ $\mathcal{E}(u(t))$ are *conserved*.
- To rigorous justify energy conservation, we need the additional asusmption that w and u are *spherically symmetric* (only a mathematical issue or there is a physical meaning?)
- We want to use mass and energy conservation to find global in time solution (condensation is preserved also for large times).

Assume that $w \geqslant 0$ (repulsive interaction). Then

$$\|u(t)\|^2_{H^1_\alpha} \approx \mathcal{M}(u(t)) + < -\Delta_\alpha u, u >$$
$$\leq \mathcal{M}(u(t)) + \mathcal{E}(u(t)) = \mathcal{M}(f) + \mathcal{E}(f)$$

Hence $\|u(t)\|_{H^1_\alpha}$ remains bounded, and by the blow-up alternatives the solution is global.

- Also if $\widehat{w} \geqslant 0$ (physical meaning of this condition?) the potential energy is positive, whence global in time solution.
- In general, for attractive w, the dynamic is more complicated: blow-up solutions, bound states.
- It would be interesting to investigate the nature of these solutions, and how they depends on the presence of a point interaction.

Thank you for your attention

Geometry of Quantum Mechanics in complex projective spaces

Davide Pastorello

Department of Mathematics, University of Trento
Trento Institute for Fundamental Physics and Application

Trieste, 18 May 2018

With the support of:

FONDAZIONE CARITRO
CASSA DI RISPARMIO DI TRENTO E ROVERETO

Geometrization of Quantum Mechanics

Describing quantum systems in terms of **geometric structures.**

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Why?

- Standard formulation of Quantum Mechanics presents a mathematical structure that is linear and algebraic (operators in Hilbert spaces)
- Classical Mechanics can be mathematically formulated in a broad and elegant differential geometric framework (symplectic manifolds, Hamiltonian fields, Poisson structures...).

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Phylosophical goal: A unified quantum/classical geometric scenario!

Technical goal: Application of powerful geometric tools that are well-known in Classical Mechanics to quantum problems.

Geometrization of Quantum Mechanics

Some landmarks

- **T. W. B. Kibble** *Geometrization of quantum mechanics*, Comm. Math. Phys. 65 (1979)
- **A. Ashtekar and T. A. Schilling** *Geometry of quantum mechanics*, AIP Conf. Proc. 342 (1995)
- **D.C. Brody and L.P. Hughston** *Geometric quantum mechanics*, J. Geom. Phys. 38 (2001)
- **J. Clemente-Gallardo and G. Marmo** *Basics of quantum mechanics, geometrization and some applications to quantum information*, Int. J. Geom. Methods Mod. Phys. 5(6) (2008)

Summary

Geometric Hamiltonian formulation of QM

- Quantum Mechanics in a classical-like fashion
- From operators to phase space functions

Geometry and quantum control

- Notions of quantum controllability
- Differential geometry and quantum controllability

Classical tools

Phase space

A classical system with n spatial degrees of freedom is described in a $2n$-**dimensional symplectic manifold** $(\mathcal{M}, \omega)$.

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A point $x = (q^1, ..., q^n, p_1, ..., p_n)$

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Phase space

A classical system with n spatial degrees of freedom is described in a $2n$-**dimensional symplectic manifold** $(\mathcal{M}, \omega)$.

Physical state

A point $x = (q^1, \dots, q^n, p_1, \dots, p_n)$

Dynamics

A curve in $(a, b) \ni t \mapsto x(t) \in \mathcal{M}$ satisfying Hamilton equations:

$$\frac{dx}{dt} = X_H(x(t))$$

$H : \mathcal{M} \to \mathbb{R}$ is the *Hamiltonian function*.
X_H is the *Hamiltonian vector field*, given by: $\omega_x(X_H, \cdot) = dH_x(\cdot)$

Classical tools

Statistical description

The state is a $\mathcal{C}^1$-function ρ on $\mathcal{M}$ and dynamics is described by the Liouville equation

$$\frac{\partial \rho}{\partial t} + \{\rho, H\}_{PB} = 0$$

Poisson bracket: $\{f, g\}_{PB} := \omega(X_f, X_g)$.

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Physical quantities are real smooth function on $\mathcal{M}$:
The Observable C^*-algebra is:

$$\mathcal{A} = \mathcal{C}^\infty(\mathcal{M})$$

Classical expecation value of $f : \mathcal{M} \to \mathbb{R}$ on ρ:

$$\langle f \rangle_\rho = \int_\mathcal{M} f(x)\rho(x) d\mu(x)$$

QM in a classical-like fashion

Standard formulation of QM in a Hilbert space $\mathcal{H}$:

Quantum states: $D(\mathcal{H}) = \{\sigma \in \mathfrak{B}_1(\mathcal{H}) | \sigma \geq 0, tr(\sigma) = 1\}$
Quantum observables: Self-adjoint operators in $\mathcal{H}$.

Pure states (extreme points of the convex set D) are in bijective correspondence with projective rays in $\mathcal{H}$:

$$\mathcal{P}(\mathcal{H}) = \frac{\mathcal{H}}{\sim} \qquad \psi \sim \phi \; \Leftrightarrow \exists \alpha \in \mathbb{C} \setminus \{0\} \; s.t. \; \psi = \alpha\phi$$

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$\dim\mathcal{H} = n < +\infty$
$\mathcal{P}(\mathcal{H})$ is a real $(2n-2)$-dimensional manifold with the following characterization of tangent space:

$p \in \mathcal{P}(\mathcal{H})$: $\forall v \in T_p\mathcal{P}(\mathcal{H}) \; \exists A_v \in \mathfrak{H}(\mathcal{H})$ s.t. $v = -i[A_v, p]$.

$\mathfrak{H}(\mathcal{H})$ is the space of hermitian operators on $\mathcal{H}$.

$\mathcal{P}(\mathcal{H})$ as a Kähler manifold

Symplectic form: $\omega_p(u,v) := -i\,k\,tr([A_u, A_v]p) \qquad k>0$
Riemannian metric:

$$g_p(u,v) := -k\,tr(([A_u,p][A_v,p]+[A_v,p][A_u,p])p) \qquad k>0$$

Almost complex form: $j_p : T_p\mathcal{P}(\mathcal{H}) \ni v \mapsto i[v,p] \in T_p\mathcal{P}(\mathcal{H})$
$p \mapsto j_p$ is smooth and $j_p j_p = -id$ for any $p \in \mathcal{P}(\mathcal{H})$:

$$\omega_p(u,v) = g_p(u, j_p v)$$

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Quantum observables as phase space functions

$$\mathcal{O} : \mathfrak{H}(\mathcal{H}) \ni A \mapsto f_A : \mathcal{P}(\mathcal{H}) \to \mathbb{R}$$

Equivalence Hamilton/Schrödinger dynamics:

$$\frac{dp}{dt} = -i[H, p(t)] \quad \Leftrightarrow \quad \frac{dp}{dt} = X_{f_H}(p(t))$$

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$$\omega_p(u,v) = g_p(u, j_p v)$$

Quantum states as Liouville densities

$$\mathcal{S} : D(\mathcal{H}) \ni \sigma \mapsto \rho_\sigma : \mathcal{P}(\mathcal{H}) \to [0,1]$$

Equivalence quantum/classical expectation values:

$$\langle A \rangle_\rho = \mathrm{tr}(A\sigma) = \int_{\mathcal{M}} f_A(p)\rho_\sigma(p)d\mu(p)$$

From operators to functions

Definition

A map $f : \mathcal{P}(\mathcal{H}) \to \mathbb{C}$ is called **frame function** if there is $W_f \in \mathbb{C}$ s.t.

$$\sum_{p \in N} f(p) = W_f$$

for any $N \subset \mathcal{P}(\mathcal{H})$ s.t. $d_g(p_1, p_2) = \frac{\pi}{2}$ for $p_1, p_2 \in \mathcal{P}(\mathcal{H})$ with $p_1 \neq p_2$ and N is maximal w.r.t. this property.

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$$\mathcal{F}^2(\mathcal{H}) := \{f : \mathcal{P}(\mathcal{H}) \to \mathbb{C} |\, f \in \mathcal{L}^2(\mathcal{P}(\mathcal{H}), \mu),\ f \text{ is a frame function}\}$$

Theorem (V. Moretti, D.P. 2014)

Phase space functions describing quantum observables are real functions in $\mathcal{F}^2(\mathcal{H})$ and obtained from operators by:

$$\mathcal{O} : \mathfrak{H}(\mathcal{H}) \ni A \mapsto f_A \qquad f_A(p) = k\, tr(Ap) + \frac{1-k}{n} tr(A) \quad k > 0$$

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Theorem (Ashtekar et al. 1995)

A vector field X on $\mathcal{P}(\mathcal{H})$ is the Hamiltonian vector field of a quantum observable (i.e. $X(p) = -i[A, p]$ with $A \in \mathfrak{H}(\mathcal{H})$) if and only if

$$\mathcal{L}_X g = 0$$

C*-algebra of quantum observables in terms of functions

$\mathcal{O} : \mathfrak{H}(\mathcal{H}) \ni A \mapsto f_A$ $\quad$ – linear extension → $\quad$ $\mathcal{O} : \mathfrak{B}(\mathcal{H}) \to \mathcal{F}^2(\mathcal{H})$

$\mathcal{F}^2(\mathcal{H})$ as C*-algebra of observables

-) Involution: $A = \mathcal{O}(f)$, $A^* = \mathcal{O}(\bar{f})$;
-) $\star$ - product: $f \star g = \mathcal{O}\left(\mathcal{O}^{-1}(f)\mathcal{O}^{-1}(g)\right)$:

$$f \star h = \frac{i}{2}\{f, h\}_{PB} + \frac{1}{2}G(df, dh) + f \cdot h \qquad k = 1$$

-) Norm: $|||f||| = \| \mathcal{O}^{-1}(f) \|$

$$|||f||| = \frac{1}{k}\left\|f - \frac{1-k}{n}\int_{\mathcal{P}(\mathcal{H})} f \; d\mu\right\|_{\infty} \qquad k > 0$$

where $d\mu$ is the volume form induced by g.

Quantum control

Controlled n-level quantum system

$$i\hbar\frac{d}{dt}|\psi\rangle = \left[H_0 + \sum_{i=1}^{m} H_i u_i(t)\right]|\psi(t)\rangle \qquad (*)$$

with initial condition $|\psi(0)\rangle = |\psi_0\rangle$.

Pure state controllability

The n-level system is **pure state controllable** if for every pair $|\psi_0\rangle, |\psi_1\rangle \in \mathcal{H}$ there exixst controls $u_1, \ldots, u_m$ and $T > 0$ such that the solution $|\psi\rangle$ of $(*)$ satisfies

$$|\psi(T)\rangle = |\psi_1\rangle$$

.

Quantum control

Controlled n-level quantum system

$$i\hbar \frac{d}{dt} U(t) = \left[H_0 + \sum_{i=1}^{m} H_i u_i(t) \right] U(t) \qquad (**)$$

with initial condition $U(0) = \mathbb{I}$.

Complete controllability

The n-level system is **complete controllable** if for any unitary operator $U_f \in U(n)$ there exist controls $u_1, \ldots, u_n$ and $T > 0$ such that the solution U of $(**)$ satisfies

$$U(T) = U_f$$

Differential geometry and quantum controllability

Geometric Hamiltonian formulation

$$\dot{p}(t) = X_0(p(t)) + \sum_{i=1}^{m} X_i(p(t))u_i(t)$$

X_i are the Hamiltonian fields on $\mathcal{P}(\mathcal{H})$ defined by the classical-like Hamiltonians obtained with our prescription.

Accessibility algebra

The smallest Lie subalgebra $\mathcal{C}$ of the Lie algebra of smooth vector fields on $\mathcal{P}(\mathcal{H})$ containing the fields $X_0, \ldots, X_m$.

Accessibility distribution

$$\mathcal{C}(p) := \mathrm{span}\{X(p) \,|\, X \in \mathcal{C}\}$$

Theorem (D.P. 2016)

A quantum system is pure state controllable if and only if the following condition is satisfied:

$$T_p\mathcal{P}(\mathcal{H}) = span\{X(p)|X \in \mathcal{C}\}$$

for some $p \in \mathcal{P}(\mathcal{H})$.

The proof is based on this proposition:

$$A \in \mathcal{L} \quad \Longleftrightarrow \quad X_{f_{-iA}} \in \mathcal{C}$$

where $\mathcal{L}$ is the Lie algebra generated by $-iH_0, ..., -iH_1$.

Corollary

A quantum system is completely controllable if and only if

$$\mathcal{C} = \mathfrak{Kill}(\mathcal{P}(\mathcal{H}))$$

An example

Consider a controlled 4-level quantum system whose dynamical Lie algebra $\mathcal{L}$ is given by the matrices of the form:

$$A = \begin{pmatrix} -ia & c & z & d \\ e & ib & f & w \\ -\overline{z} & d & ia & e \\ f & -\overline{w} & c & -ib \end{pmatrix},$$

where $a, b, c, d, e, f \in \mathbb{R}$ and $z, w \in \mathbb{C}$.

An example

Consider a controlled 4-level quantum system whose dynamical Lie algebra $\mathcal{L}$ is given by the matrices of the form:

$$A = \begin{pmatrix} -ia & c & z & d \\ e & ib & f & w \\ -\bar{z} & d & ia & e \\ f & -\overline{w} & c & -ib \end{pmatrix},$$

where $a, b, c, d, e, f \in \mathbb{R}$ and $z, w \in \mathbb{C}$.
Let $p = diag(1, 0, 0, 0)$ and calculate:

$$X_A(p) = \begin{pmatrix} 0 & -c & -z & -d \\ e & 0 & 0 & 0 \\ -\bar{z} & 0 & 0 & 0 \\ f & 0 & 0 & 0 \end{pmatrix},$$

$\dim \mathcal{C}(p) = 6 = \dim T_p\mathcal{P}(\mathcal{H})$. Pure state controllability!

References

- **V. Moretti and D.P.** *Generalized spherical harmonics, frame functions and Gleason theorem*. Annales Henri Poincaré v. 2013, 14, n.5 (2013)
- **D.P.** *A geometric hamiltonian description of composite quantum systems and quantum entanglement*. Int. J. Geom. Methods in Mod. Phys. v. 12, n. 7 (2015)
- **V. Moretti and D.P.** *Frame functions in finite-dimensional quantum mechanics and its hamiltonian formulation on complex projective spaces*.Int. J. Geom. Methods Mod. Phys. Vol. 13, No. 02, 1650013. (2016)
- **D.P.** *Geometric Quantum Mechanics and applications*. Int. J. Geom. Methods Mod. Phys., 13, 1630017 (2016)
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Thank you for your attention!

Quantum model for Impulsive Stimulated Raman Scattering (ISRS)

Stefano Marcantoni

University of Trieste

Trieste Junior Quantum Days
May 18, 2018

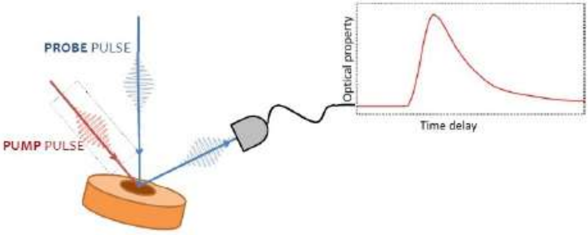
Outline

INCEPT

INhomogenieties and fluctuations in quantum CohErent Phases by ultrafast optical Tomography

- Experiments: Prof. Daniele Fausti (P.I.),
  Theory: Prof. Fabio Benatti
- Ultrashort dynamics in complex materials (sub-picosecond time scales)
- Cross-fertilization between quantum optics (quantum state tomography) and condensed matter physics (pump-probe experiments)

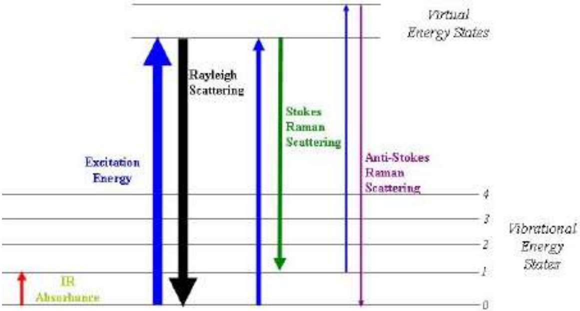
Pump-probe experiments



- Pump pulse excites the material
- Probe pulse (less intense) test the evolution after a delay time t

Raman scattering

- Raman scattering is a kind of inelastic scattering for light
- One photon loses energy exciting one phonon in the material (Stokes process)... or
  One photon increases its energy destroying one phonon (anti-Stokes process)



Probe-target interaction

- Initial state (probe + target): $|\alpha\rangle\langle\alpha| \otimes \varrho_t$
- Refraction at the boundary

$$H_{ref} = \sum_{j,\mu,\mu'} \left(\eta^{(0)}_{\mu\mu'} + \langle b + b^\dagger\rangle_t\, \eta^{(1)}_{\mu\mu'}\right)\left(a^\dagger_{\mu j}\, r_{\mu' j} + a_{\mu j}\, r^\dagger_{\mu' j}\right)$$

- Raman scattering

$$H_{Ram} \;:=\; \sum_{\mu,\mu'} \chi_{\mu,\mu'}\left[\left(\sum_j a^\dagger_{\mu j} a_{\mu' j+\frac{\Omega}{\delta}}\right) b^\dagger + \left(\sum_j a_{\mu j} a^\dagger_{\mu' j+\frac{\Omega}{\delta}}\right) b\right],$$

$a_{\mu j}, r_{\mu j}, b$ are bosonic operators

Dynamics and observables

- Evolution operator

$$U(\tau) = U_{bulk}(\tau)\, U_{ref}$$

$$U_{ref} = \exp(-i\, H_{ref})\ , \quad U_{bulk}(\tau) = \exp(-i\tau\, H_{Raman}) \qquad (\hbar = 1)\ ,$$

- Average of an observable X_{phot}

$$\langle X_{phot}(\tau)\rangle_t = \mathrm{Tr}[U(\tau)\,|\alpha\rangle\langle\alpha| \otimes \varrho_t\, U^\dagger(\tau)\, X_{phot}]$$

- Coherent state of the probe

$$|\alpha\rangle = \exp\left(\sum_j \alpha_{xj} a^\dagger_{xj} - \alpha^*_{xj} a_{xj}\right)|0\rangle\ , \quad a_{xj}|\alpha\rangle = \alpha_{xj}\ , \quad a_{yj}|\alpha\rangle = 0\ ,$$

$$\alpha_{xj} = \exp\left(-\frac{(j\delta)^2}{2\sigma^2}\right) \mathrm{e}^{i\varphi}$$

Pump-target interaction

- Same Hamiltonian for the light-matter interaction but different approximation
- Mean field for photons $a_{\lambda j} \to \alpha^P_{\lambda j}$
- Explicit dependence on the polarization angle (with respect to x)

$$\alpha^P_{xj} = \alpha^P_{0j} \cos(\theta_P), \quad \alpha^P_{yj} = \alpha^P_{0j} \sin(\theta_P)$$

- Phononic operator shifted $\langle b \rangle_t = \mathrm{Tr}(\varrho_t b) = \mathrm{Tr}(\varrho b_t)$

$$\begin{aligned} b \to b_t =& U^\dagger_{ref}\, U^\dagger_{bulk}(\tau)\, U^\dagger_{free}(t)\, b\, U_{free}(t)\, U_{bulk}(\tau)\, U_{ref} \\ \simeq& \, \mathrm{e}^{-i\Omega t} \left(b - i\tau \sum_{j,\lambda\lambda'} \chi_{\lambda\lambda'}\, \alpha^{P*}_{\lambda j}\, \alpha^P_{\lambda' j + \frac{\Omega}{\delta}} \right) . \end{aligned}$$

Assumptions on the interaction (good for Quartz)

- Zeroth order refraction matrix

$$\eta^{(0)} = \begin{pmatrix} \eta_{xx}^{(0)} & \eta_{xy}^{(0)} \\ \eta_{xy}^{(0)} & \eta_{xx}^{(0)} \end{pmatrix}$$

- First order refraction matrix depending on the phonon involved (same for χ)

$$A : \eta^{(1)} = \begin{pmatrix} \eta_{xx}^{(1)} & 0 \\ 0 & \eta_{xx}^{(1)} \end{pmatrix}, \quad E_L : \eta^{(1)} = \begin{pmatrix} \eta_{xx}^{(1)} & 0 \\ 0 & -\eta_{xx}^{(1)} \end{pmatrix},$$
$$E_T : \eta^{(1)} = \begin{pmatrix} 0 & \eta_{xy}^{(1)} \\ \eta_{xy}^{(1)} & 0 \end{pmatrix}$$

Geometry

- Phonons selected by the angle between the polarization of the pump and the x axis (θ_P)

$$\begin{aligned} A :\langle b\rangle_t &= C_A \mathrm{e}^{-i\Omega_A t - i\pi/2} \\ E_L :\langle b\rangle_t &= C_E \cos(2\theta_P)\, \mathrm{e}^{-i\Omega_E t - i\pi/2} , \\ E_T :\langle b\rangle_t &= C_E \sin(2\theta_P)\, \mathrm{e}^{-i\Omega_E t - i\pi/2} , \end{aligned}$$

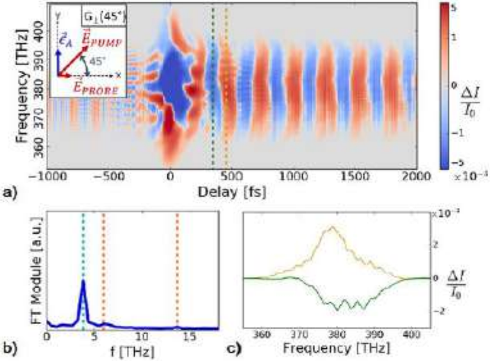
Remember:

$$\begin{aligned} b \to & U^\dagger_{ref}\, U^\dagger_{bulk}(\tau)\, U^\dagger_{free}(t)\, b\, U_{free}(t)\, U_{bulk}(\tau)\, U_{ref} \\ & \simeq \mathrm{e}^{-i\Omega t} \left(b - i\tau \sum_{j,\lambda\lambda'} \chi_{\lambda\lambda'}\, \alpha^{P*}_{\lambda j}\, \alpha^{P}_{\lambda' j + \frac{\Omega}{\delta}} \right) . \end{aligned}$$

Mode occupation numbers (y polarization)

$$\begin{aligned}\langle N_{yk}(\tau)\rangle_t \simeq & \langle b+b^\dagger\rangle_t |\alpha_{xk}|^2 \, F^y_{ref} \\ & - i\,\langle b^\dagger - b\rangle_t |\alpha_{xk}| \left(|\alpha_{xk+\frac{\Omega}{\delta}}| - |\alpha_{xk-\frac{\Omega}{\delta}}|\right) \, F^y_{Ram}(\tau)\end{aligned}$$

$$\begin{aligned} A: \quad & F^y_{ref} = 0, && F^y_{Ram}(\tau) = 0, \\ E_L: \quad & F^y_{ref} = 0, && F^y_{Ram}(\tau) = 0, \\ E_T: \quad & F^y_{ref} = 2\eta^{(1)}_{xy}\eta^{(0)}_{xy}\sin^2(\eta^{(0)}_{xx}), && F^y_{Ram}(\tau) = 0. \end{aligned}$$

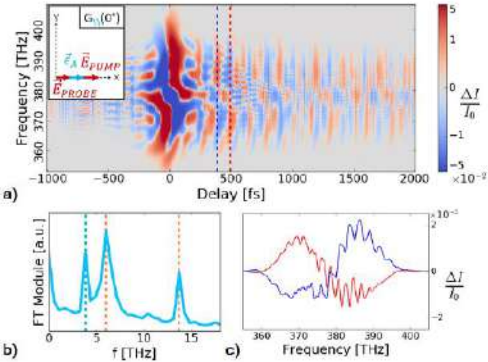


- Pump at 45°: A and E_T phonons excited
- Orthogonal polarization (leading term): E_T Refractive

Mode occupation numbers (x polarization)

$$\langle N_{xk}(\tau)\rangle_t \simeq \cos^2(\eta_{xx}^{(0)})|\alpha_{xk}|^2 + \langle b+b^\dagger\rangle_t|\alpha_{xk}|^2\, F_{ref}^x \\ - i\,\langle b^\dagger - b\rangle_t|\alpha_{xk}|\left(|\alpha_{xk+\frac{\Omega}{\delta}}| - |\alpha_{xk-\frac{\Omega}{\delta}}|\right)\, F_{Ram}^x(\tau)$$

$$\begin{aligned} A:&\quad F_{ref}^x = -\eta_{xx}^{(1)}\sin(2\eta_{xx}^{(0)}), & F_{Ram}^x(\tau) &= \chi_{xx}\tau\cos^2(\eta_{xx}^{(0)}),\\ E_L:&\quad F_{ref}^x = -\eta_{xx}^{(1)}\sin(2\eta_{xx}^{(0)}), & F_{Ram}^x(\tau) &= \chi_{xx}\tau\cos^2(\eta_{xx}^{(0)}),\\ E_T:&\quad F_{ref}^x = -2\eta_{xy}^{(1)}\eta_{xy}^{(0)}\cos^2(\eta_{xx}^{(0)}), & F_{Ram}^x(\tau) &= 0. \end{aligned}$$

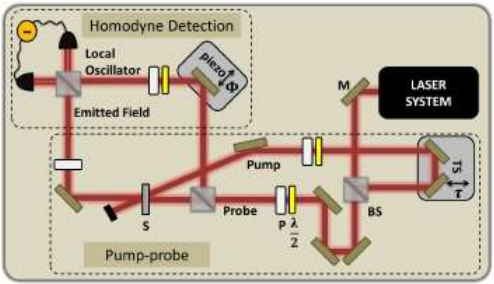


- Pump at 0°: A and E_L phonons excited
- Parallel polarization: Raman and Refractive effects are both visible

Results Experiment 1 (summary)

- Phase mismatch between Raman and refractive modulation
- Selection of different phonons according to the polarization of the pump
- Different behaviour of Raman and refractive modulation depending on the phonon involved and on the polarization selected by the analyzer
- Good agreement between theory and experiment

Quadrature: Homodyne detection + Time-resolved spectroscopy



- We combine two different experimental techniques to probe the nonequilibrium response of the material

Average quadrature

- Measured quantity: Current difference I

$$I = \sum_j \left(c^\dagger_{xj} c_{xj} \; - \; d^\dagger_{xj} d_{xj} \right) , \qquad c_j = \frac{a_{xj} + a^{LO}_{xj}}{\sqrt{2}} \,,\, d_j = \frac{a_{xj} - a^{LO}_{xj}}{\sqrt{2}} \,.$$

- Quadrature:

$$X_s = \frac{1}{\sqrt{2}} \sum_j \left(a_{xj}\, z^*_j\, \mathrm{e}^{-i\Phi_j(s)} \; + \; a^\dagger_{xj}\, z_j\, \mathrm{e}^{i\Phi_j(s)} \right) \propto I$$

- Theoretical prediction:

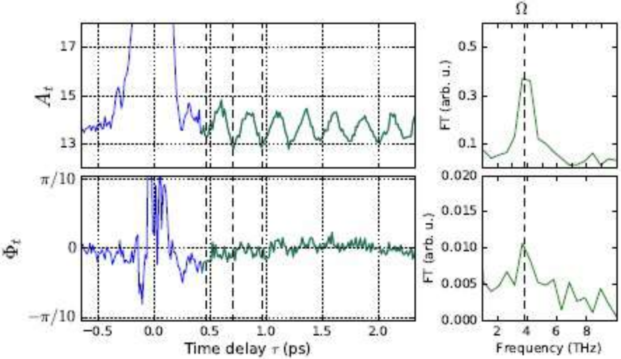
$$\langle X_s(\tau) \rangle = \mathcal{A}_t \cos(\omega_0 s + \Phi_t)$$

where

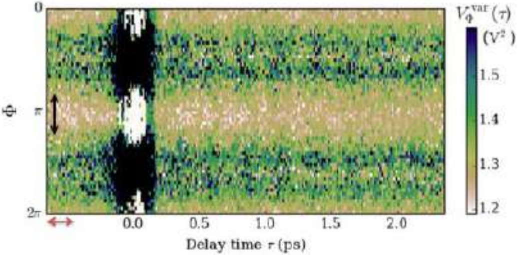
$$\mathcal{A}_t \simeq \mathcal{A}\,(1 + \overline{\eta}\sin(\Omega t))\,, \quad \Phi_t \simeq 2\overline{\chi}\sin(\Omega t).$$

Average quadrature

$$\mathcal{A}_t \simeq \mathcal{A}\left(1 + \overline{\eta}\sin(\Omega t)\right), \quad \Phi_t \simeq 2\overline{\chi}\sin(\Omega t).$$



Variance of the quadrature: work in progress



- For a coherent initial state: variance is time-independent up to second order in the coupling
- Higher order effects or (more likely) signature of a statistical mixture

Conclusions and Outlook

Results:

- Fully quantum model for Impulsive Stimulated Raman Scattering (ISRS)
- Outcomes of two different experiments correctly reproduced

Future work:

- Complete tomography of the state of light (variance of the quadrature)
- Role of quantum correlations
- More interesting (complex) dynamics in the sample (*e.g.* interaction between the vibrational and electronic degrees of freedom)

Thank you for your attention!

Social dinner: Pizzeria "Al Barattolo" at 20.00.