Entanglement and thermodynamics in out-of-equilibrium systems

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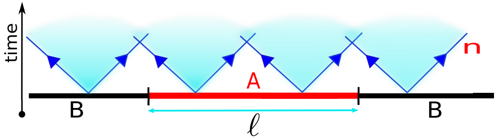
Trieste, 18/05/2018



Outline

[V.A. and P. Calabrese, PNAS **114**, 7947 (2017)]

- **Complexity** of out-of-equilibrium quantum matter.
- **Entanglement** and **quenches**.
- **Goal: Entanglement dynamics after quantum quenches.**
- **Semiclassical** picture & **Integrability**.
- **von Neumann** vs **Rényi** entropies.

[V. Alba and P. Calabrese, Phys. Rev. B 96, 11541 (2017)]

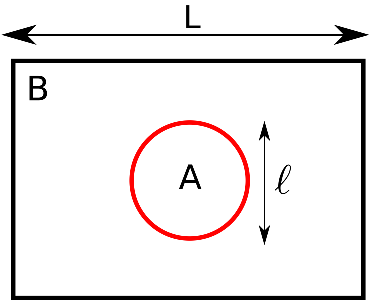
[V. Alba and P. Calabrese, J. Stat. Mech. (2017) 113105]

[V. Alba and P. Calabrese, arXiv:1712.07529]

[V. Alba, arXiv:1706.00020]

Out-of-equilibrium **isolated** many-body systems

- **Question**: How do simple descriptions (**thermodynamics**) emerge in out-of-equilibrium **isolated** sytems?



$A \cup B =$ **isolated** universe

Unitary dynamics under Hamiltonian H

$L, \ell \to \infty, \text{with}\, \ell \ll L$

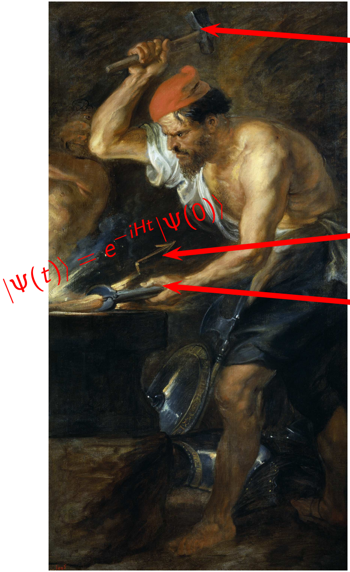
$time \to \infty$

- Long-time limit of local reduced density matrix? Is it **thermal?**

$$\rho_A \equiv \mathrm{Tr}_B \rho_{A \cup B}$$

Wonders of out-of-equilibrium systems

P. P. Rubens, *Vulcan forging the Thunderbolts of Jupiter* (1637), Prado Museum



"sudden" global manipulation

Quantum quench, Floquet dynamics, adiabatic quench (ramping)

isolated quantum system (T=0)

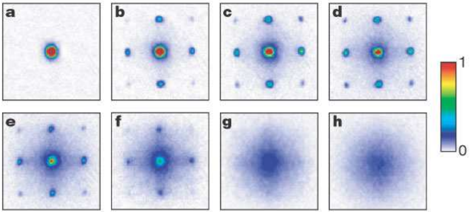
Theory toolbox:

- Integrability
- DMRG
- Exact diagonalization
- Field Theory methods

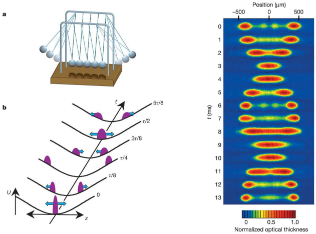
Challenge: No **unifying** theory framework

Out of equilibrium physics in cold-atom experiments

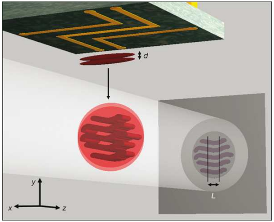
[Greiner, Nature (2002)]



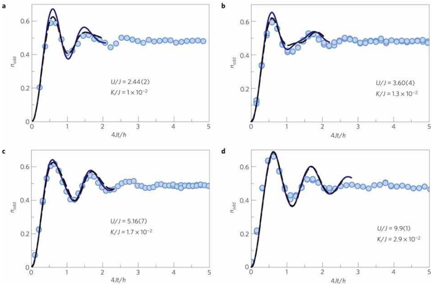
[Kinoshita et al., Nature 440, 900 (2006)]



[Hofferberth, Nature (2007)]



[Trotzky, Nature Phys. (2012)]



Quantum quench protocol

- Initial state $|\Psi_0\rangle$ $\Rightarrow$ **unitary** evolution under a many-body Hamiltonian $\mathcal{H}$

$\{|\psi_\alpha\rangle\}$ eigenstates of $\mathcal{H}$

$$|\Psi_0\rangle = \sum_\alpha c_\alpha |\psi_\alpha\rangle \qquad |\Psi(t)\rangle = \sum_\alpha e^{iE_\alpha t} c_\alpha |\psi_\alpha\rangle$$

$$c_\alpha \equiv \langle \Psi_0 | \psi_\alpha \rangle$$

- For a generic observable $\widehat{\mathcal{O}}$:

$$\langle \Psi(t)|\widehat{\mathcal{O}}|\Psi(t)\rangle = \sum_{\alpha,\beta} e^{i(E_\alpha - E_\beta)t} c^*_\alpha c_\beta \widehat{\mathcal{O}}_{\alpha\beta}$$

- **Long time** $\Rightarrow$ **diagonal ensenble**.

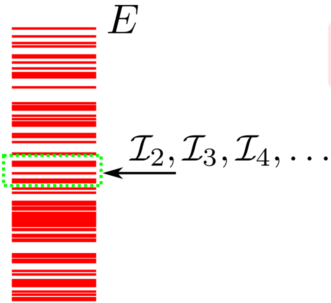
$$\overline{\langle \Psi(t)|\widehat{\mathcal{O}}|\Psi(t)\rangle} = \langle \hat{\mathcal{O}} \rangle_{DE} = \sum_\alpha |\langle \Psi_0 | \psi_\alpha \rangle|^2 \widehat{\mathcal{O}}_{\alpha\alpha}$$

Equilibration in integrable models

- **Integrability** $\Rightarrow$ **Local** (quasi-local) **conserved** quantities $\mathcal{I}_j$.

$$[\mathcal{H}, \mathcal{I}_j] = 0,\ \forall j \quad \text{and} \quad [\mathcal{I}_j, \mathcal{I}_k] = 0,\ \forall j,k \qquad \mathcal{I}_2 \equiv \mathcal{H}$$

- Include extra charges in Gibbs $\Rightarrow$ **Generalized Gibbs Ensemble** (GGE).



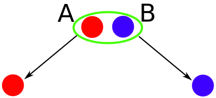
$$\rho^{GGE} = \frac{1}{Z} \exp\left(\sum_j \beta_j \mathcal{I}_j \right)$$

[Jaynes, 1957;Rigol,2008]

- **Generalized microcanonical** principle.

- Einstein-Podolsky-Rosen paradox:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle) + |\downarrow\uparrow\rangle$$



- Perfect anticorrelated spin measurements.

RESEARCH ARTICLE

QUANTUM OPTICS

Satellite-based entanglement distribution over 1200 kilometers

Juan Yin,[1,2] Yuan Cao,[1,2] Yu-Huai Li,[1,2] Sheng-Kai Liao,[1,2] Liang Zhang,[2,3] Ji-Gang Ren,[1,2] Wen-Qi Cai,[1,2] Wei-Yue Liu,[1,2] Bo Li,[1,2] Hui Dai,[1,2] Guang-Bing Li,[1,2] Qi-Ming Lu,[1,2] Yun-Hong Gong,[1,2] Yu Xu,[1,2] Shuang-Lin Li,[1,2] Feng-Zhi Li,[1,2] Ya-Yun Yin,[1,2] Zi-Qing Jiang,[3] Ming Li,[3] Jian-Jun Jia,[3] Ge Ren,[4] Dong He,[4] Yi-Lin Zhou,[5] Xiao-Xiang Zhang,[6] Na Wang,[7] Xiang Chang,[8] Zhen-Cai Zhu,[5] Nai-Le Liu,[1,2] Yu-Ao Chen,[1,2] Chao-Yang Lu,[1,2] Rong Shu,[2,3] Cheng-Zhi Peng,[1,2*] Jian-Yu Wang,[2,3*] Jian-Wei Pan[1,2*]

Long-distance entanglement distribution is essential for both foundational tests of quantum physics and scalable quantum networks. Owing to channel loss, however, the previously achieved distance was limited to ~100 kilometers. Here we demonstrate satellite-based distribution of entangled photon pairs to two locations separated by 1203 kilometers on Earth, through two satellite-to-ground downlinks with a summed length varying from 1600 to 2400 kilometers. We observed a survival of two-photon entanglement and a violation of Bell inequality by 2.37 ± 0.09 under strict Einstein locality conditions. The obtained effective link efficiency is orders of magnitude higher than that of the direct bidirectional transmission of the two photons through telecommunication fibers.

Science 356, 1140 (2017)

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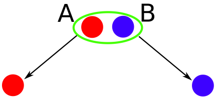

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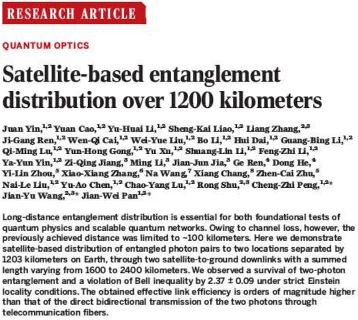
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- Haiku view on entanglement:

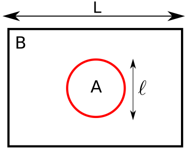
Up here down there, these bonds are stronger than time. N.B.

Entanglement entropy in many-body systems

- Consider a quantum system in d dimensions in a **pure** state $|\Psi\rangle$

$$\rho \equiv |\Psi\rangle\langle\Psi|$$

- If the system is **bipartite**:



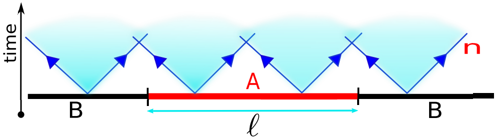
$$H = H_A \otimes H_B \rightarrow \rho_A = Tr_B \rho$$

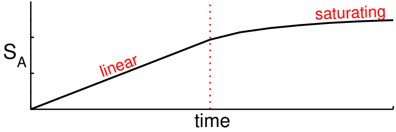
- How to quantify the entanglement (**quantum** correlations) between A and B?
  - **von Neumann** entropy $S_A = -Tr \rho_A \log \rho_A = -\sum_i \lambda_i \log \lambda_i$
  - **Rényi** entropies $S_A^{(n)} = -\frac{1}{n-1} \log(Tr \rho_A^n) = -\frac{1}{n-1} \log(\sum_i \lambda_i^n)$

Entanglement dynamics: Semiclassical picture

- **Extensive** amount of energy $\Rightarrow$ **quasi-particles** produced uniformly in the initial state.

[Calabrese, Cardy, 2005]





$$S_A(t) \propto 2t \int_{2|v|t<\ell} d\lambda v(\lambda) f(\lambda) + \ell \int_{2|v|t>\ell} d\lambda f(\lambda)$$

- Requires quasi-particles **group velocities** $v(\lambda)$
- $f(\lambda)$ **cross-section** for quasi-particle production.
- Exact for free models.

[Fagotti, Calabrese, 2008]

Integrable models (*à la* Bethe ansatz)

- **Integrability** $\Rightarrow$ **stable** families of "**single particle**" excitations.

$$\lambda_{n,j} = \text{particle quasimomentum} \approx \textbf{rapidity}.$$

- Generic eigenstate:

$$|\{\lambda_{n,j}\}\rangle$$

- Thermodynamic limit $\Rightarrow$ **macrostate** $\Rightarrow$ **particle** and **hole** densities

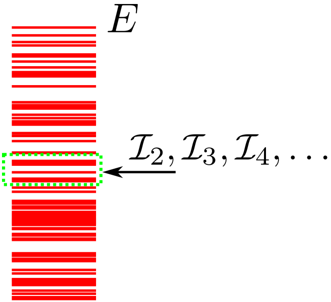
$$|\{\rho_n(\lambda), \rho_n^{(h)}(\lambda)\}\rangle$$

- \# equivalent microscopic eigenstates $\Rightarrow$ Yang-Yang **entropy**

$$S_{YY} \equiv L \sum_n \int d\lambda [\rho_n^{(t)} \log \rho_n^{(t)} - \rho_n \log \rho_n - \rho_n^{(h)} \log \rho_n^{(h)}]$$

Quenches in integrable models

- Key idea: **Steady state** $\Rightarrow$ **macrostate $|\rho_n\rangle$**.



$e^{S[\rho_n]} =$ **# of representative eigenstates**

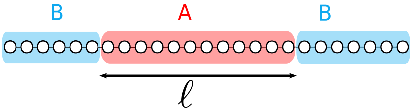
$S[\rho_n] =$ **thermodynamic entropy**

$|\rho_n\rangle$

- **Integrability** $\Rightarrow$ $|\rho_n\rangle$ and $S[\rho_n]$ can be determined **analytically**.

Steady state entanglement entropy

- Steady-state **entanglement** entropy density is the **thermodynamic** entropy.



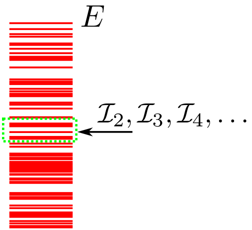
$$S_A/\ell = (\mathrm{Tr}\rho^{GGE} \log \rho^{GGE})/L = \sum_n \int d\lambda s_n(\lambda)$$

- Cross section for quasi-particle production is fixed $f(\lambda) = s_n(\lambda)$:

$$S_A(t) \xrightarrow{t\to\infty} \ell \sum_n \int d\lambda s_n(\lambda)$$

Entangling quasi-particles

- How to identify the **entangling** quasi-particles?

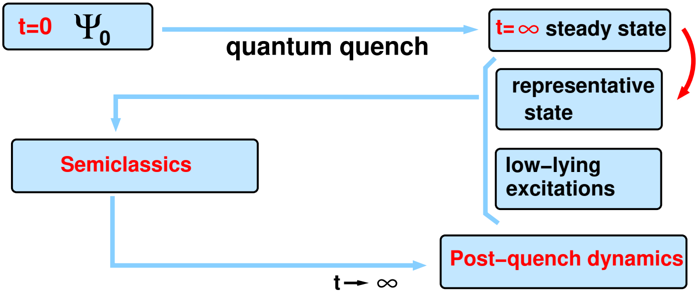


$|\rho_n\rangle$

[J.-S. Caux and F. Essler, Phys. Rev. Lett. 110, 257203 (2013)]

- **Local** observables $\Rightarrow$ dynamics determined by **low-lying** excitations around steady state $|\rho_n\rangle$.

Theoretical program



$$S_A(t) \propto \sum_k \left[t \int_{|v_k|t<\ell} d\lambda v_k(\lambda) s_k(\lambda) + \ell \int_{|v_k|t>\ell} d\lambda s_k(\lambda) \right]$$

- Spin-1/2 anisotropic Heisenberg (XXZ) chain.

$$\mathcal{H}_{XXZ} = \sum_{i=1}^{L} (S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+ + \Delta S_i^z S_{i+1}^z) \qquad \Delta \geq 1$$

- Initial states:

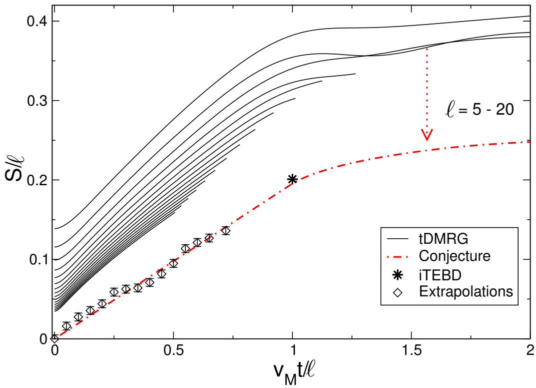
Tilted ferromagnet $|UP, \vartheta\rangle \equiv \frac{1}{\sqrt{2}} e^{i\vartheta/2 \sum_j \sigma_j^y} |\uparrow\uparrow \cdots\rangle$

Tilted Néel $|N, \vartheta\rangle \equiv \frac{1}{\sqrt{2}} e^{i\vartheta/2 \sum_j \sigma_j^y} (|\uparrow\downarrow\rangle^{\otimes L/2} + |\downarrow\uparrow\rangle^{\otimes L/2})$

Majumdar-Ghosh (Dimer) $|MG\rangle \equiv (\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}})^{\otimes L/2}$

Numerical checks: Full time evolution

- *XXZ* chain with $\Delta = 2$: Quench from Néel state.



- Fairly good agreement apart from finite size (time) corrections.

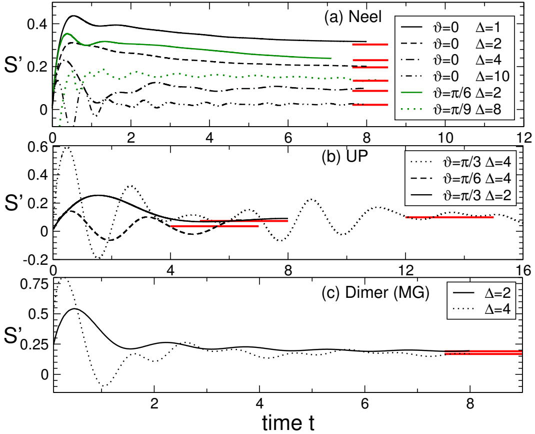
$$S_A(t) \propto \sum_k \Big[t \int_{|v_k|t<\ell} d\lambda v_k(\lambda) s_k(\lambda) + \ell \int_{|v_k|t>\ell} d\lambda s_k(\lambda) \Big]$$

Numerical checks: Linear growth

- Quench in the *XXZ* chain.

iTEBD from [Fagotti et al., 2015]

$$S' \equiv \frac{dS(v_M t)}{d(v_M t)}$$



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Conclusions

- **Entanglement** dynamics after quantum **quenches** in **integrable** models.
- Improved **Semiclassical** picture using **integrability**.
- Entanglement dynamics encoded in the **steady state** and **low-lying** excitations around it.

Thanks!