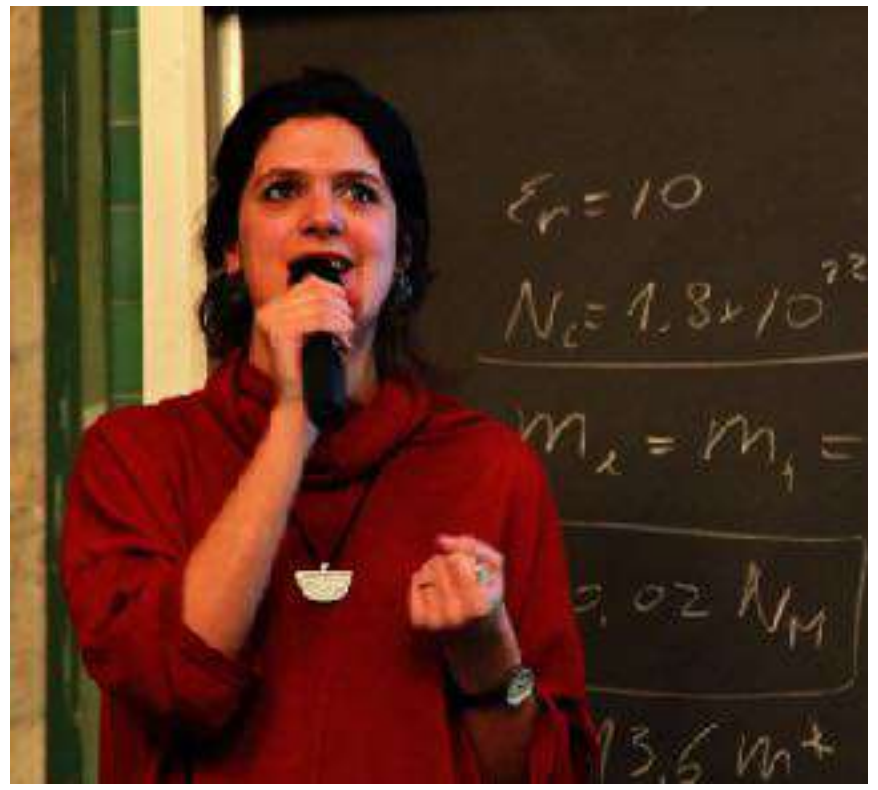
How long-range interactions slow down entanglement growth

Alessio Lerose

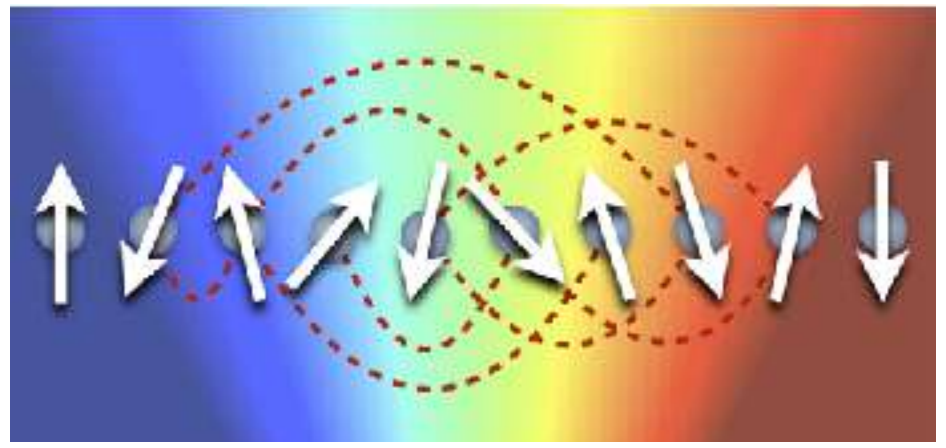
July 25, 2019 - Trieste Junior Quantum Days

joint work with Silvia Pappalardi
(also PhD student @ SISSA and ICTP, Trieste)



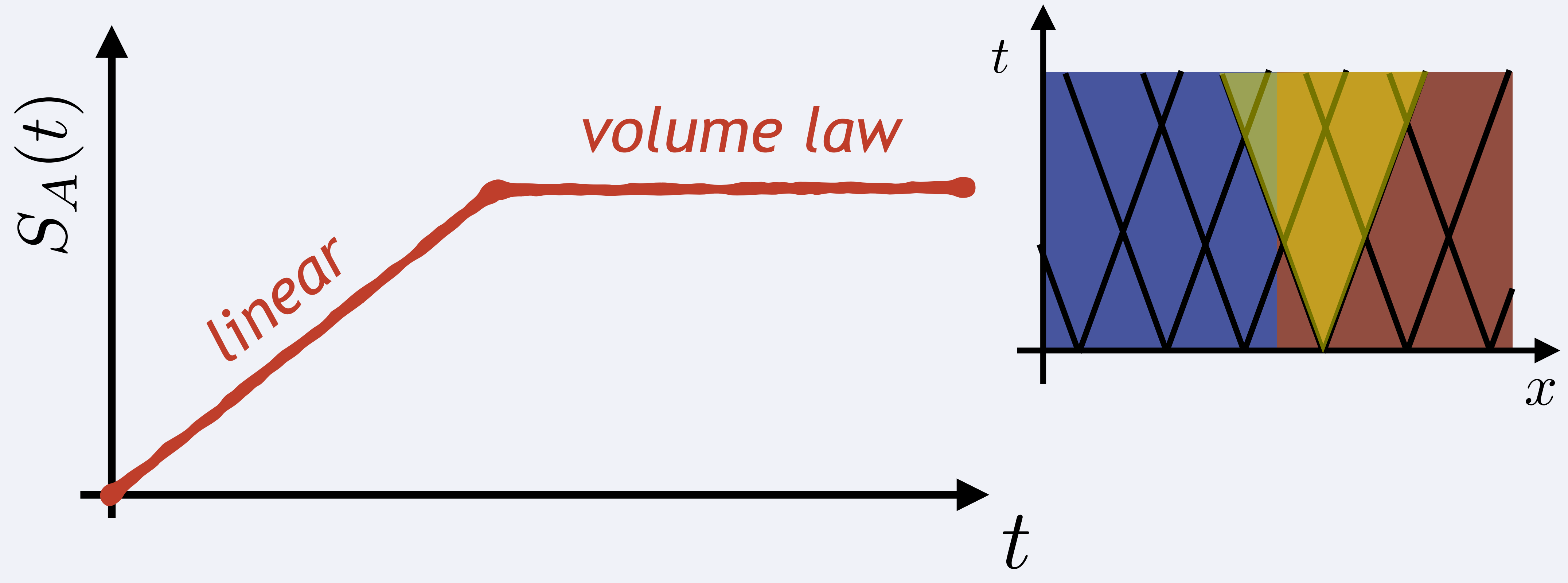
Entanglement entropy evolution

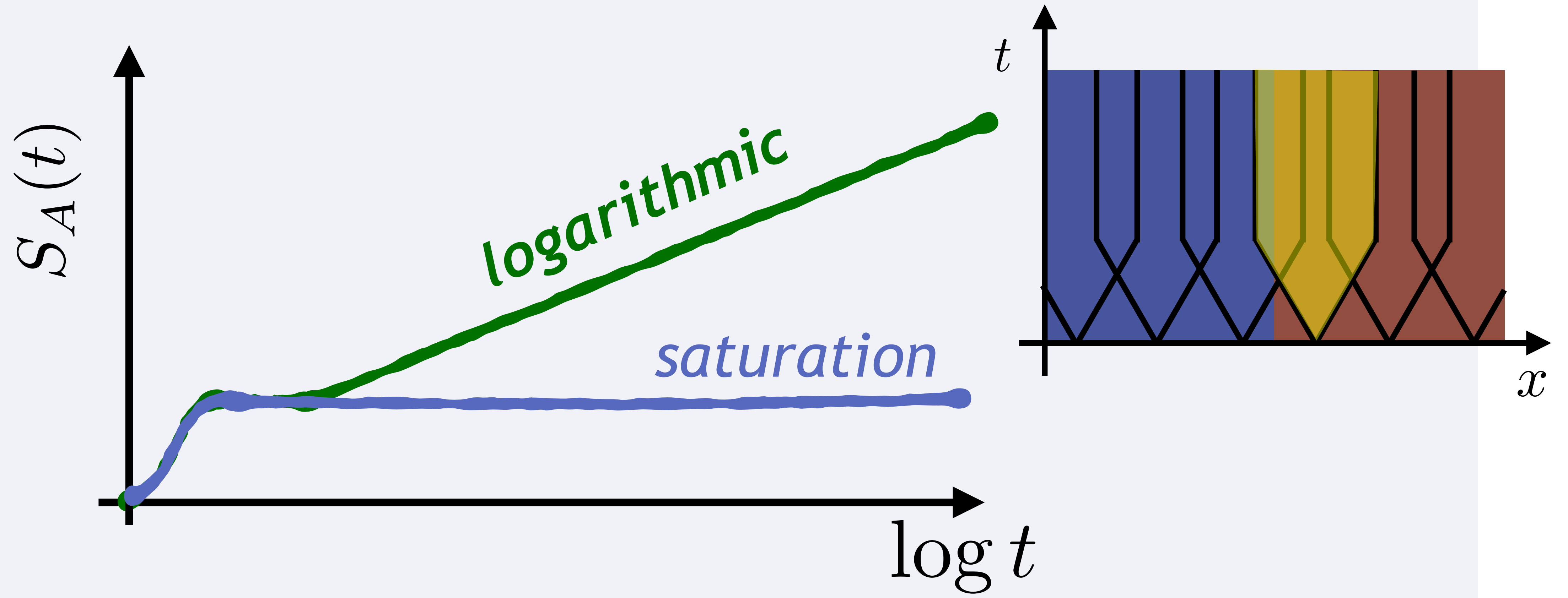
- unveils crucial properties of **quantum dynamics** and its classical **simulations** (MPS, TDVP)

$$|\psi(t)\rangle = e^{iHt}|$$  $$\rangle$$

$$S_A(t) = -\mathrm{Tr}\,\hat{\rho}_A(t)\log\hat{\rho}_A(t) \qquad \hat{\rho}_A(t) = \mathrm{Tr}_B|\psi(t)\rangle\langle\psi(t)| \qquad \mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

Paradigms with short-range interactions:





[Calabrese, Cardy - JSTAT, 2005]
[Nahum, Ruhman, Vijay, Haah - Phys. Rev. X, 2017]

[Žnidarič, Prosen, Prelovšek - Phys. Rev. B, 2008]
[Bardarson, Pollmann, Moore - Phys. Rev. Lett., 2012]
[Serbyn, Papić, Abanin - Phys. Rev. Lett., 2013]

- is very hard to measure

[Islam, Ma, Preiss, Tai, Lukin, Rispoli… - Nature, 2015]
[Elben, Vermersch, Dalmonte, Cirac, Zoller - Phys. Rev. Lett. 2018]
[Mendes-Santos, Giudici, Fazio, Dalmonte - arXiv1904.07782, 2019]
…

Long-range interactions

Classical physics:

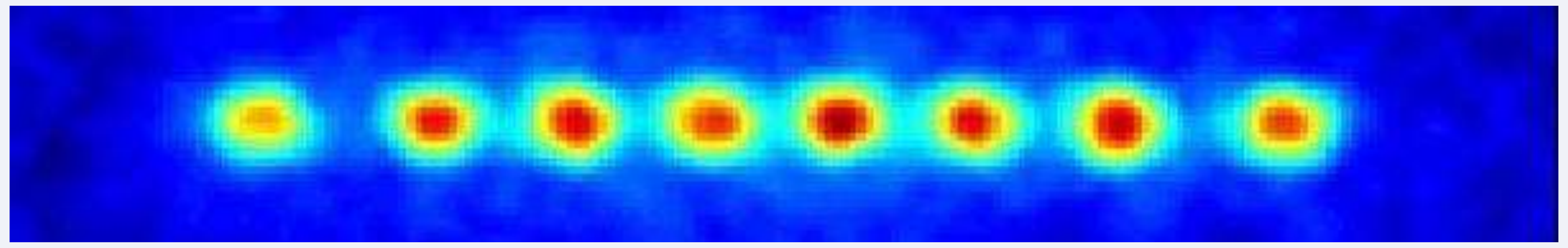
(galaxies, plasmas, ionic crystals,...)

$$J_{ij} \sim \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|^\alpha} \qquad \alpha \leq d \qquad d \text{ - } \textit{dimensional}$$

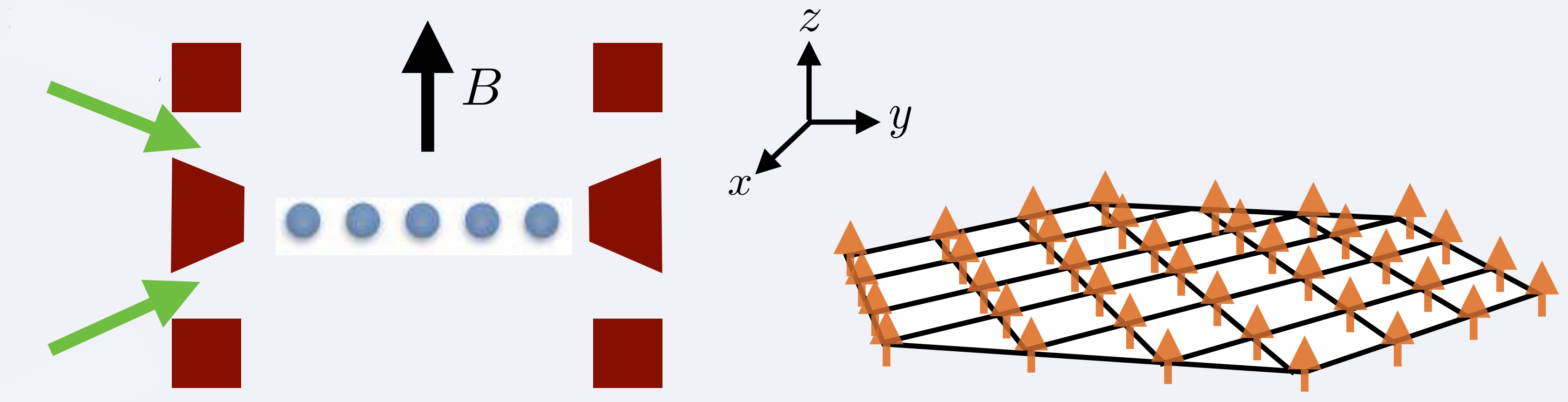
[Campa, Dauxois, Fanelli, Ruffo - UOP Oxford, 2014]

Quantum experiments in Atomic-Molecular-Optical physics:

Hyperfine levels of ultracold trapped ions:



$0.5 < \alpha < 1.8$



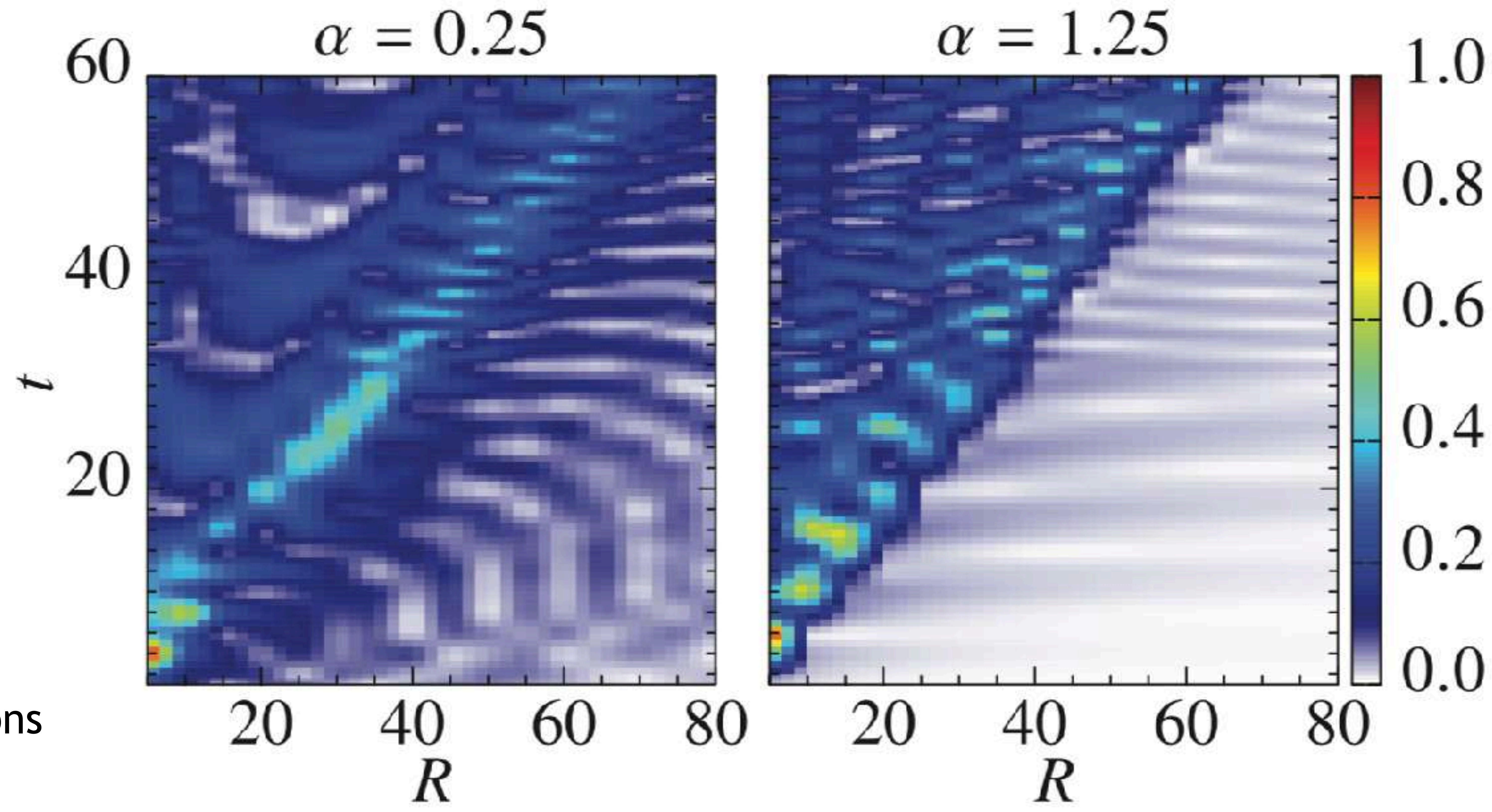
$0.02 < \alpha < 0.2$

N.B. interactions are practically instantaneous!

[Blatt, Roos - Nature Physics, 2012]
[Zhang, Pagano, Hess, ... Monroe - Nature, 2017]

[Bohnet, Sawyer, Britton, ... Bollinger - Science, 2016]

Correlation spreading with long-range interactions



Fast buildup of
long-distance correlations

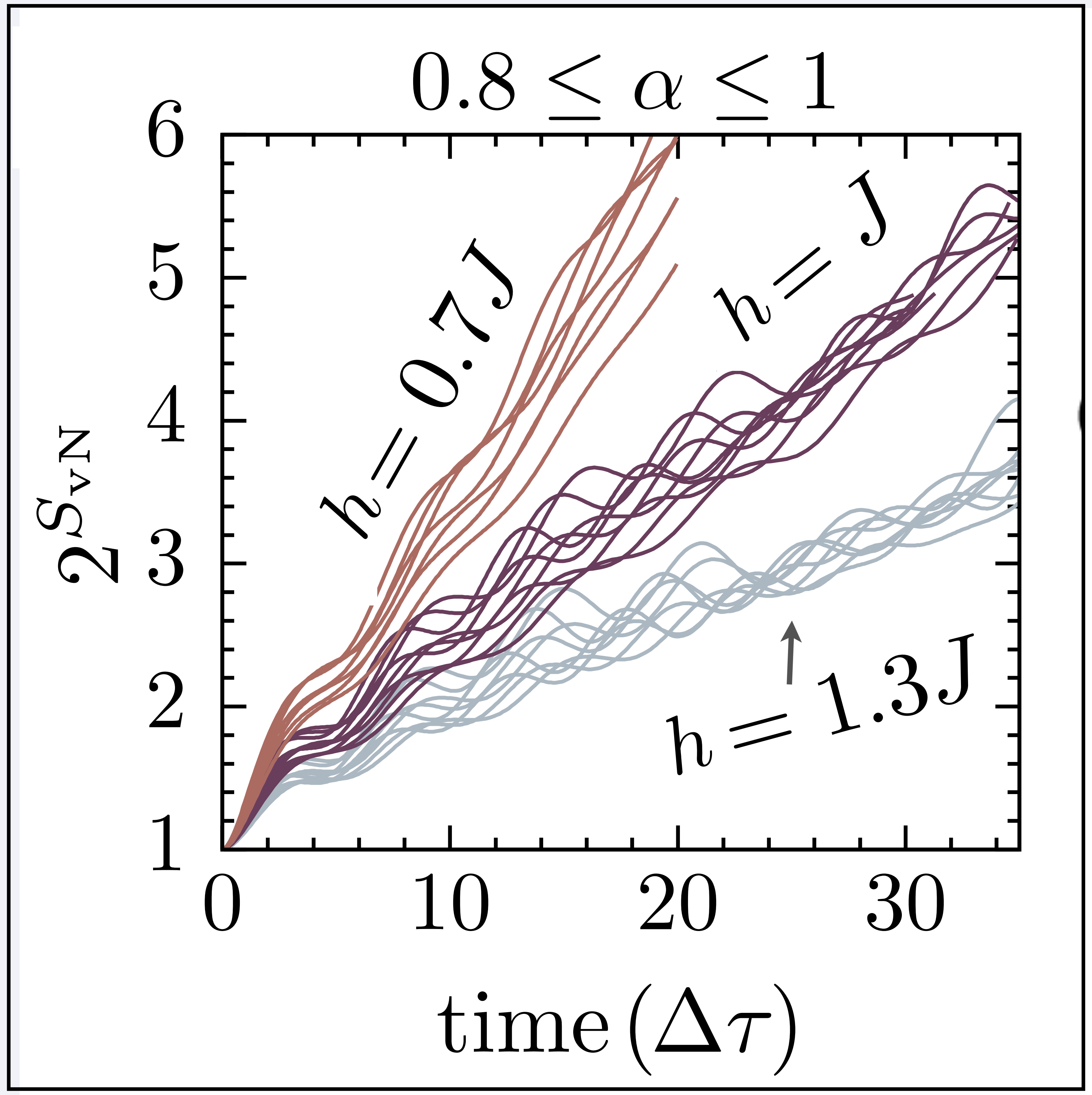
Violations of linear
light-cone spreading

Typical behavior of spatiotemporal correlations:
(from Lepori, Trombettoni, Vodola, JStat ’17)

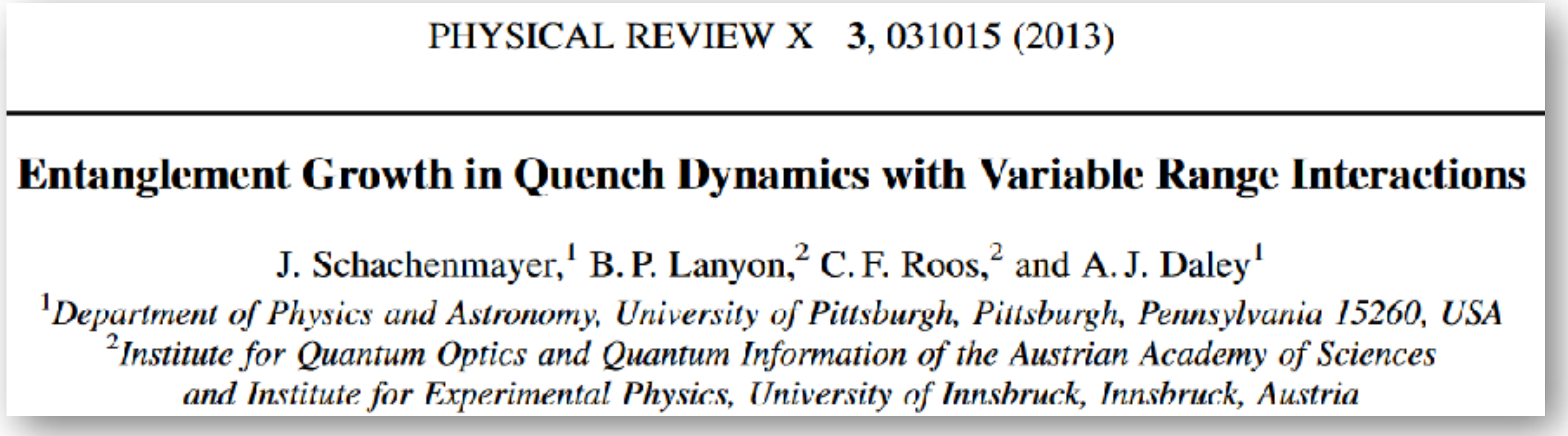
[Hauke, Tagliacozzo, Eisert, Lewenstein, Kastner, Gorshkov, Carleo-Cevolani, Wouters, Essler, Daley, Rey, Roscilde, Pupillo, Trombettoni, nakamura, Nayak, Yao,…]

Entanglement growth with long-range interactions

Quench from $|\psi_0\rangle = |\uparrow\uparrow\ldots\uparrow\rangle$ with $\hat{H} = -J\sum_{i\neq j}^{N}\frac{\hat{\sigma}_i^x\hat{\sigma}_j^x}{|i-j|^\alpha} - h\sum_i^N \hat{\sigma}_i^z$



$N = 30,\ 40,\ 50 \quad D_{\mathrm{MPS}} = 120$



PHYSICAL REVIEW X **3**, 031015 (2013)

Entanglement Growth in Quench Dynamics with Variable Range Interactions

J. Schachenmayer,[1] B. P. Lanyon,[2] C. F. Roos,[2] and A. J. Daley[1]

[1]*Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260, USA*

[2]*Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences and Institute for Experimental Physics, University of Innsbruck, Innsbruck, Austria*

different behavior. Counterintuitively, quenches above the critical point for these long-range interactions lead only to a logarithmic increase of bipartite entanglement in time, so that in this regime, long-range interactions produce a slower growth of entanglement than short-range interactions.

[Schachenmayer, Lanyon, Roos, Daley - Phys. Rev. X, 2013]

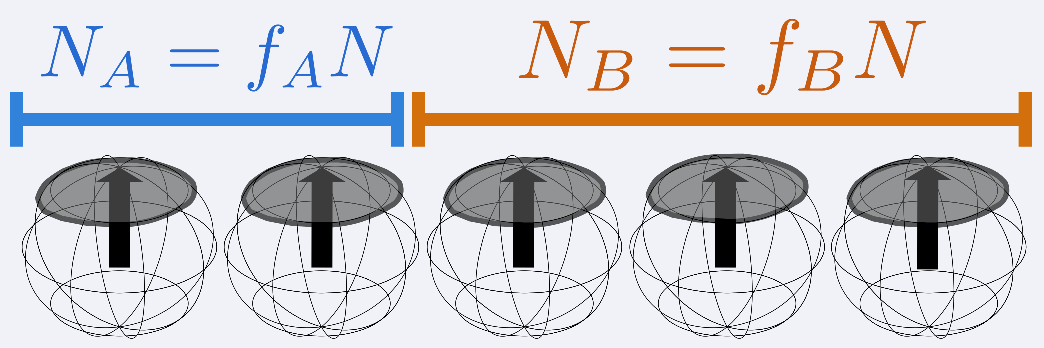
[Buyskikh, Fagotti, Schachenmayer, Essler, Daley - Phys. Rev. A, 2016]

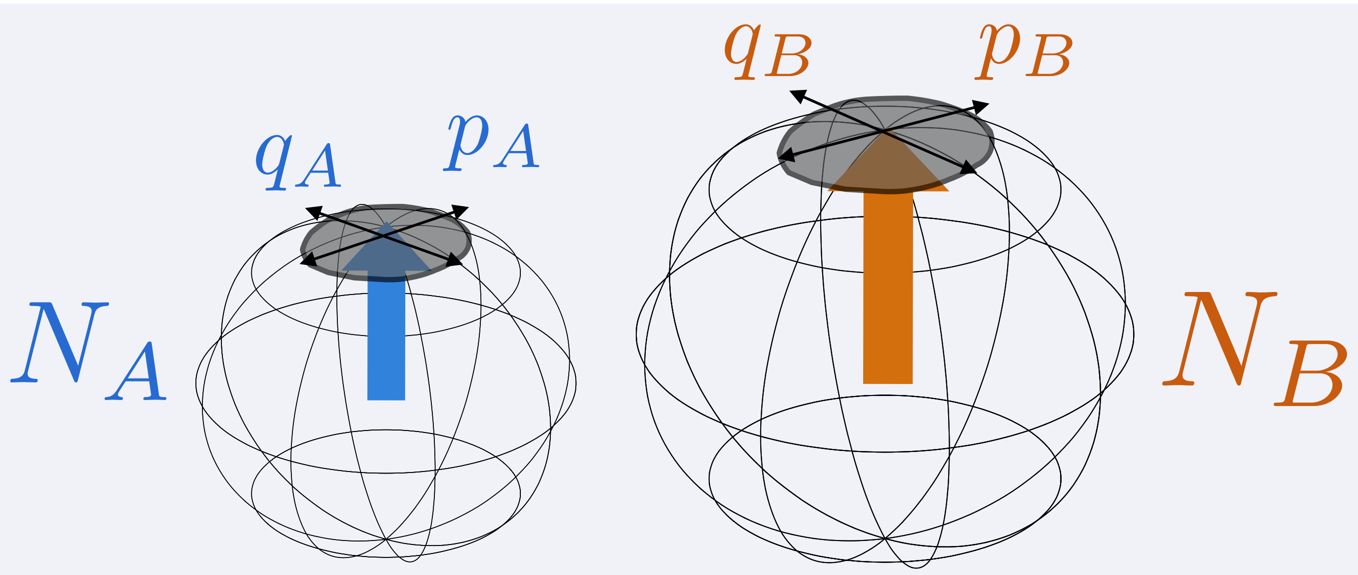
Collective dynamics $\alpha = 0$

- Hamiltonian $\tilde{\hat{H}}_0(t)$
- EX: $H_{\mathrm{LMG}} = -\frac{2J}{N}\sum_{i\neq j=1}^{N} \hat{s}_i^x \hat{s}_j^x - 2h\sum_{i=1}^{N} \hat{s}_i^z$

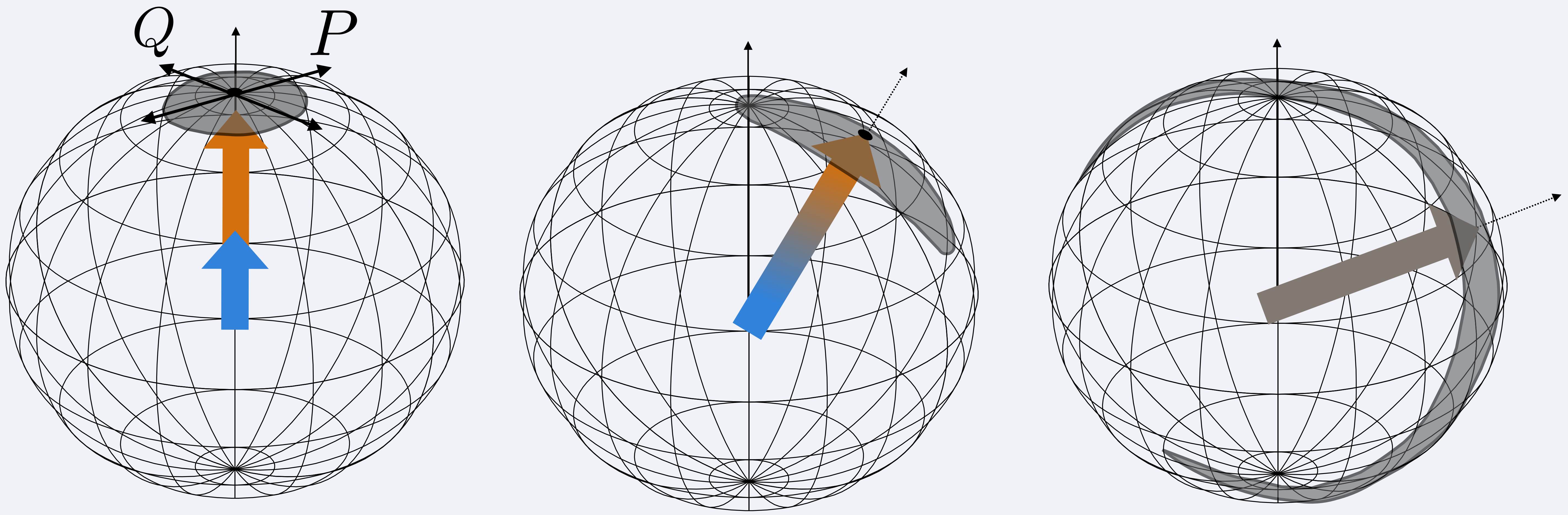
- Collective spin $\hat{\mathbf{S}} = \sum_{i=1}^{N} \hat{\mathbf{s}}_i$
- is extensive and conserved $\left[\hat{\mathbf{S}}^2, \hat{H}\right] = 0$
- ground states $S = N/2$

$N_A + N_B = N$





Classical trajectory + wavefunction squeezing



[Vidal, Dusuel, Barthel - JSTAT, 2007]

Dynamics: time-dependent Holstein-Primakoff

- decompose the collective spin $\hat{\mathbf{S}} = \hat{\mathbf{S}}_A + \hat{\mathbf{S}}_B$

- Holstein-Primakoff: treat the spins as free bosons $N \gg 1$

$$(\hat{q}_A, \hat{p}_A) \ (\hat{q}_B, \hat{p}_B) \longleftrightarrow (\hat{Q}, \hat{P})$$

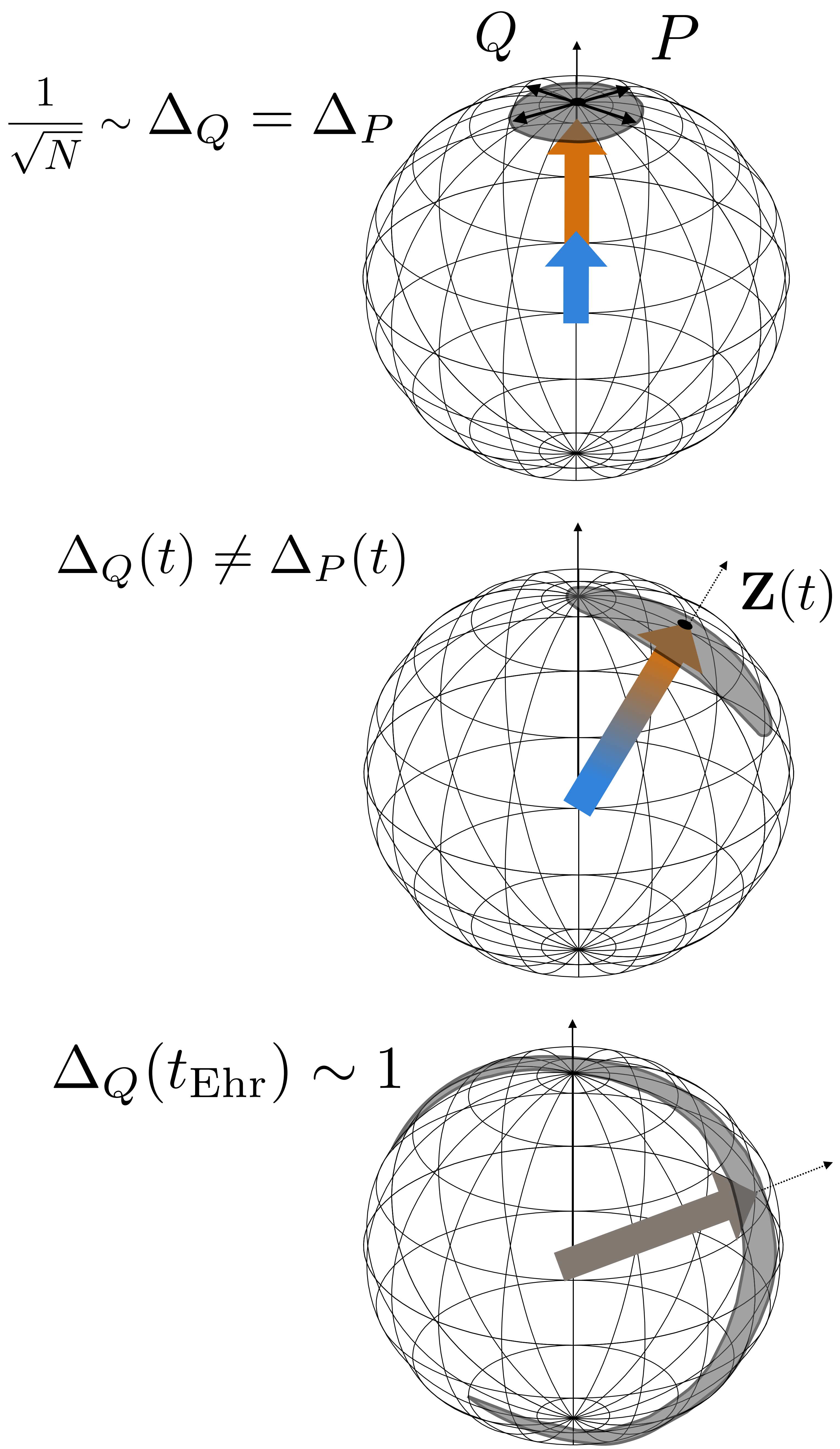
Work in a reference frame following the classical spin

$$\widetilde{\hat{H}}(t) = \hat{H} - \boldsymbol{\omega}(t) \cdot \hat{\mathbf{S}}$$

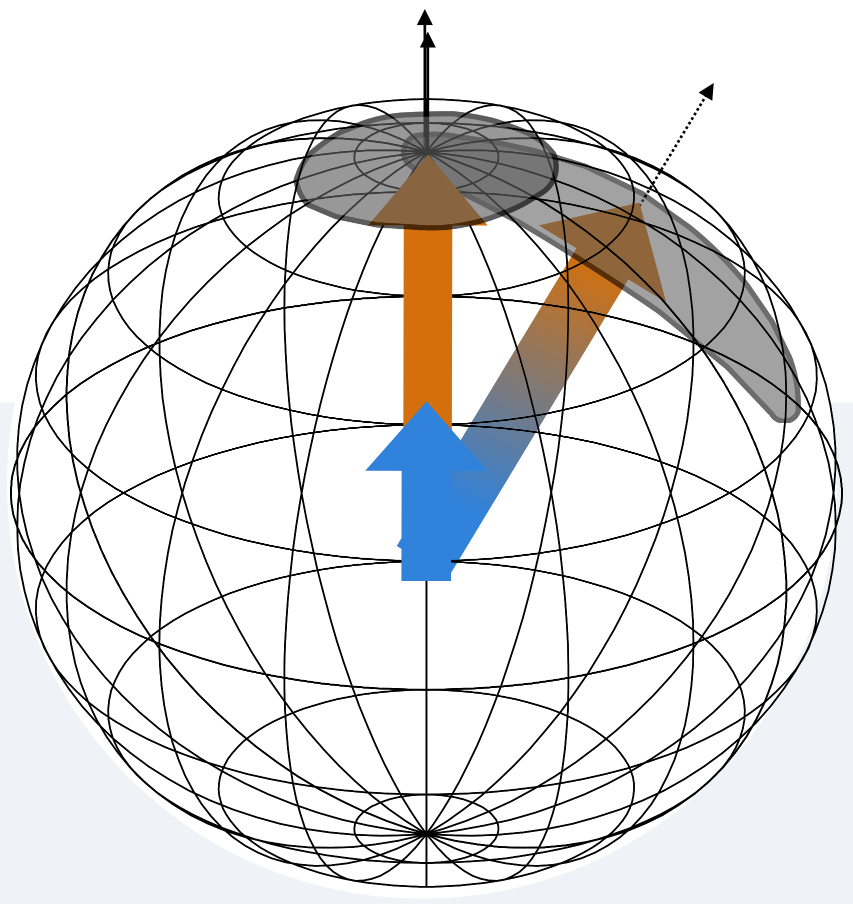
- quadratic Hamiltonian for the fluctuations

$$\widetilde{\hat{H}}(t) = h^{(2)}_{QQ}(t)\,\frac{\hat{Q}^2}{2} + h^{(2)}_{PP}(t)\,\frac{\hat{P}^2}{2} + h^{(2)}_{QP}(t)\,\frac{\hat{Q}\hat{P} + \hat{P}\hat{Q}}{2} + \mathcal{O}\left(1/\sqrt{N}\right)$$

- validity before the **Ehrenfest time**



$S_A(t)$ and collective excitations



Entanglement between bosons (q_A, p_A) and (q_B, p_B)

the system is quadratic: $\hat{\rho}_A$ is gaussian

$$G_A = \begin{pmatrix} G^{q_A q_A} & G^{q_A p_A} \\ G^{q_A p_A} & G^{p_A p_A} \end{pmatrix}$$

correlation matrix

$$\det G_A = \frac{1}{4} + f_A f_B \, \langle \hat{n}_{\text{exc}} \rangle$$

$$\hat{n}_{\text{exc}} = \frac{\hat{Q}^2 + \hat{P}^2 - 1}{2}$$

$$S_A = 2\sqrt{\det G_A} \operatorname{arccoth}\left(2\sqrt{\det G_A}\right) + \frac{1}{2}\log\left(\det G_A - \frac{1}{4}\right)$$

entangled $\langle \hat{n}_{\text{exc}} \rangle \gg 1$

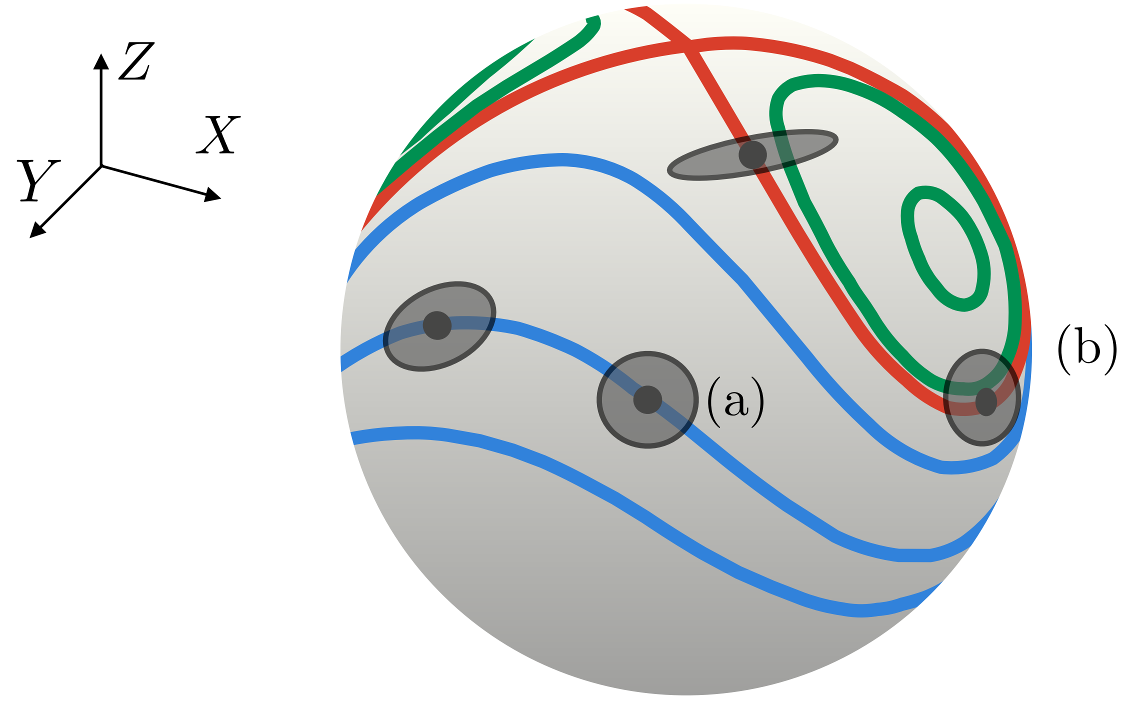
$$S_A \sim \frac{1}{2}\log\langle \hat{n}_{\text{exc}}\rangle + 1 + \frac{1}{2}\log f_A(1 - f_A)$$

separable states $\langle \hat{n}_{\text{exc}} \rangle = 0$ $\det G_A = \frac{1}{4}$ $S_A = 0$

[Barthel, Chung, Schollwock - Phys. Rev. A, 2006]

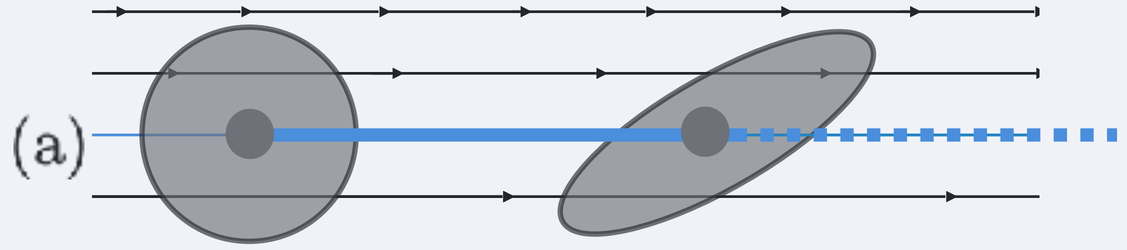
Relation to semi-classical trajectories

$$S_A(t) \sim 1 + \frac{1}{2}\log f_A f_B + \frac{1}{2}\log\langle \hat{n}_{\text{exc}}(t)\rangle$$



Start from the state $|\psi_0\rangle = |\rightarrow\rightarrow\cdots\rightarrow\rangle$ and evolve $H_{\text{LMG}} = -\frac{2J}{N}\sum_{i\neq j=1}^{N}\hat{s}_i^x\,\hat{s}_j^x - 2h\sum_{i=1}^{N}\hat{s}_i^z$

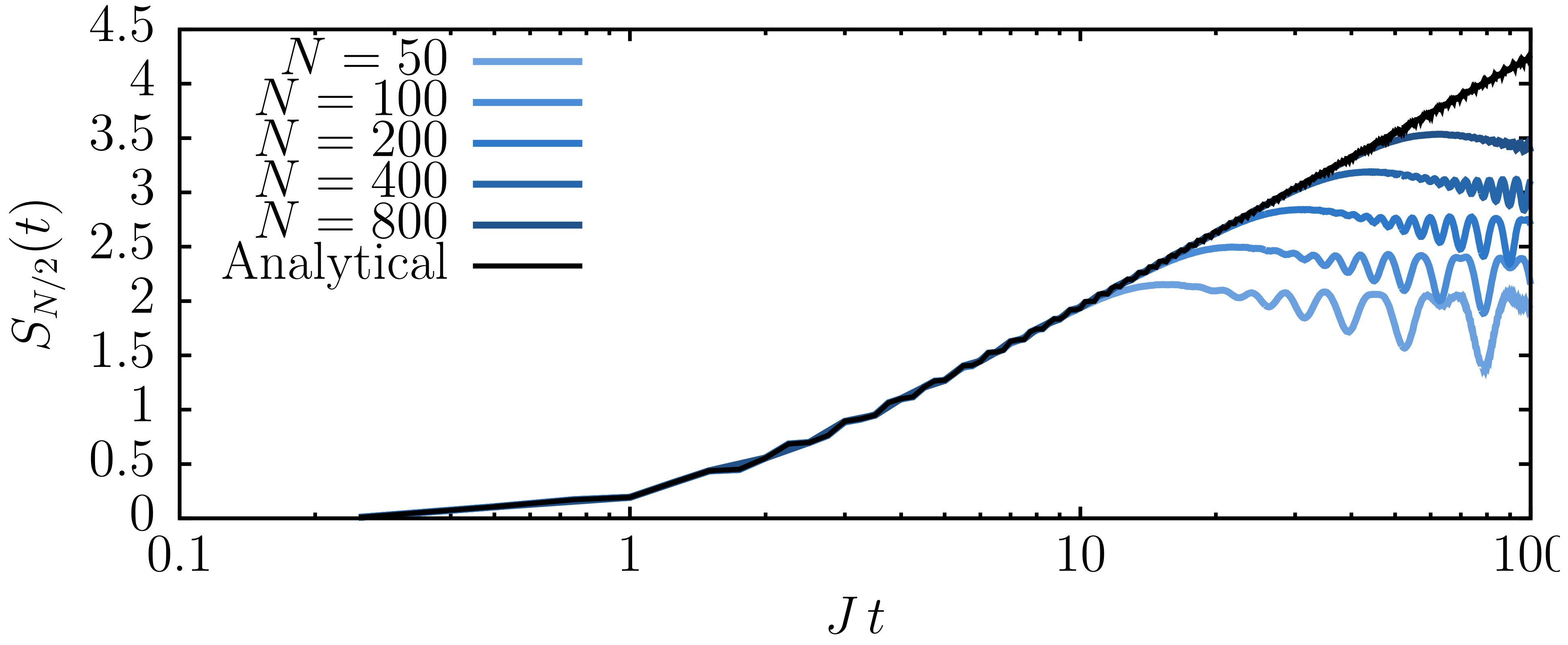
Generic quenches

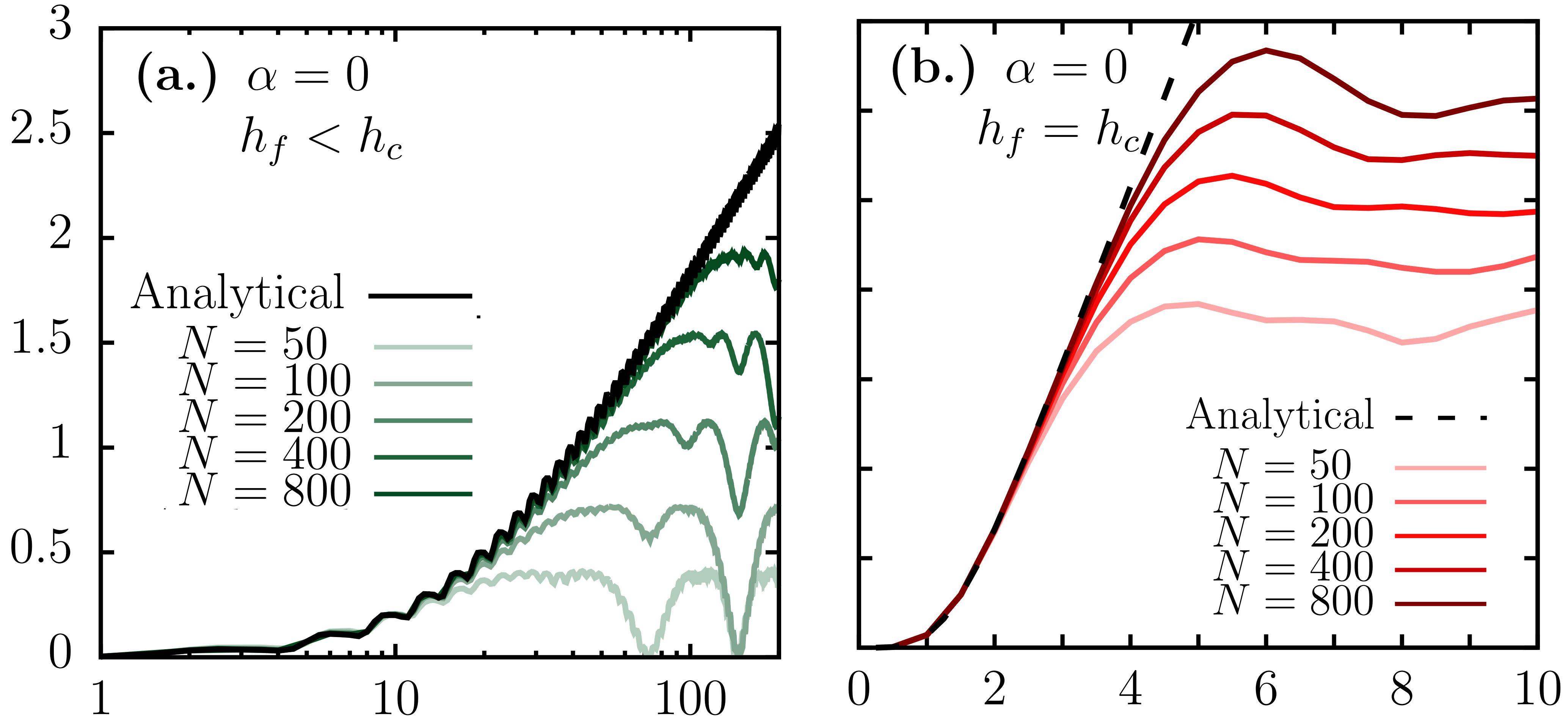


$$\langle \hat{n}_{\text{exc}}(t)\rangle \sim t^2$$

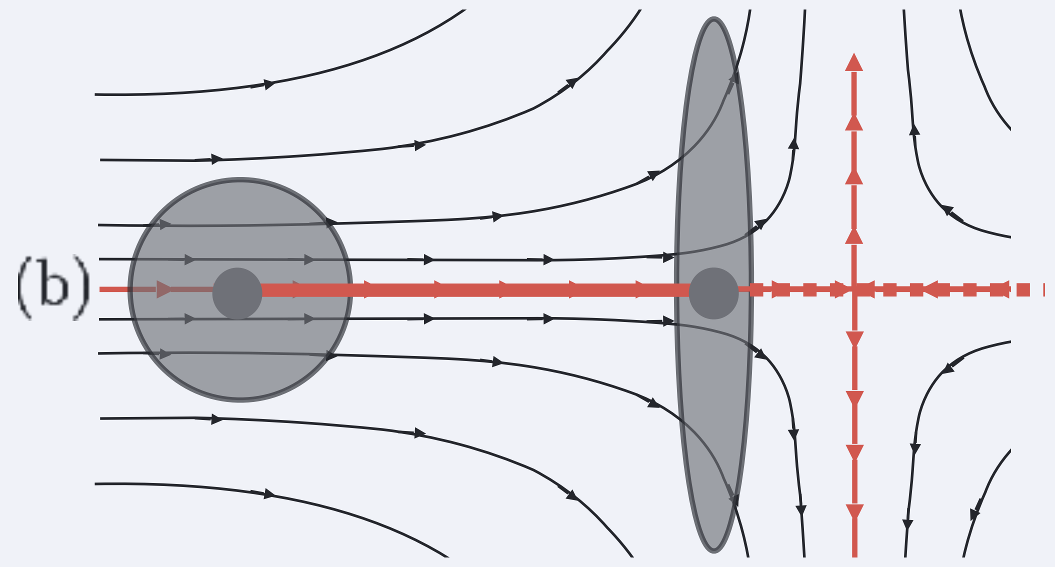
$$S_A(t) \sim \log t$$

Numerical simulations by exact diagonalization





Unstable Trajectory

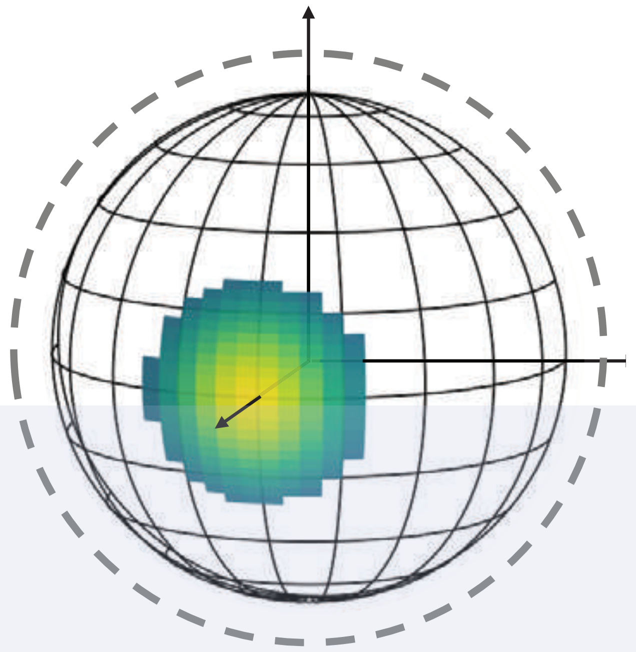


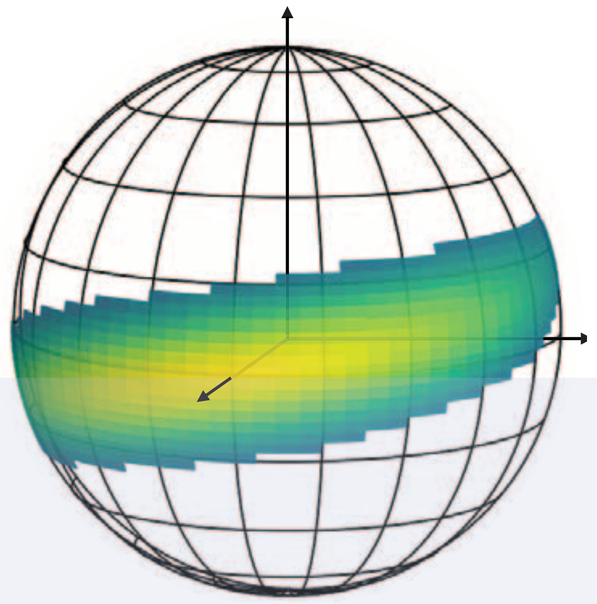
$$\langle \hat{n}_{\text{exc}}(t)\rangle \sim e^{2\lambda t}$$

$$S_A(t) \sim \lambda_{h_c}\, t$$

Interpretation

connection with **spin squeezing**





$$\xi^2 \equiv \frac{\mathrm{Min}_{\mathbf{u}\perp\mathbf{z}}\langle(\mathbf{u}\cdot\mathbf{S})^2\rangle}{N/4} = 1 + 2\langle\hat{n}_{\mathrm{exc}}\rangle - 2\sqrt{\langle\hat{n}_{\mathrm{exc}}\rangle(1+\langle\hat{n}_{\mathrm{exc}}\rangle)}$$

ξ^2	~ 1	$\sim \frac{1}{N}$
$\langle\hat{n}_{\mathrm{exc}}\rangle$	~ 0	$\sim N$

- *known witness of many-particle entanglement*
- experimentally measurable!

[Bohnet, Sawyer, Britton, ... Bollinger - Science, 2016]
[Muessel, Strobel, Joos, Nicklas, Stroescu... - Science APB, 2013]

[Kitagawa, Ueda - Phys. Rev. A, 1993]
[Wineland, Bollinger, Itano, Heinzen - Phys. Rev. A, 1994]
[Sørensen, Mølmer - Phys. Rev. Lett., 2001]
[Sørensen, Duan, Cirac, Zoller - Nature, 2001]

effective temperature

$$\hat{\rho}_{A,B} = \frac{e^{-\beta_{\mathrm{eff}}\hat{H}_{A,B}}}{Z_{A,B}}$$

Modular Hamiltonian

$$\beta_{\mathrm{eff}} = 2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{1+4f_A f_B\langle\hat{n}_{\mathrm{exc}}\rangle}}\right)$$

\`heating up' the two subsystems, continuously accumulating entanglement

Spatially decaying interactions

$$\hat{H} = -\frac{J}{\mathcal{N}_{\alpha,N}} \sum_{i\neq j} \frac{\hat{\sigma}_i^x \hat{\sigma}_j^x}{|\mathbf{r}_i - \mathbf{r}_j|^\alpha} - h \sum_i \hat{\sigma}_i^z$$

Kač normalization $\mathcal{N}_{\alpha,N} = \frac{1}{N} \sum_{i\neq j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|^\alpha}$ $\quad \mathcal{N}_{0,N} = N$

(to get a finite dynamical time scale)

Fourier representation:

$k = 0$ *collective mode*

- Time-dependent Holstein-Primakoff

$$\hat{H} = -\frac{1}{N} \sum_k \tilde{J}_k(\alpha)\, \tilde{\sigma}_k^x\, \tilde{\sigma}_{-k}^x - h\, \tilde{\sigma}_{k=0}^z$$

Lerose et al., PRL ‘18, PRB ‘19

$$\hat{\tilde{H}}(t) = \hat{\tilde{H}}_0(t) + \hat{H}_{\mathrm{sw}}(t)$$

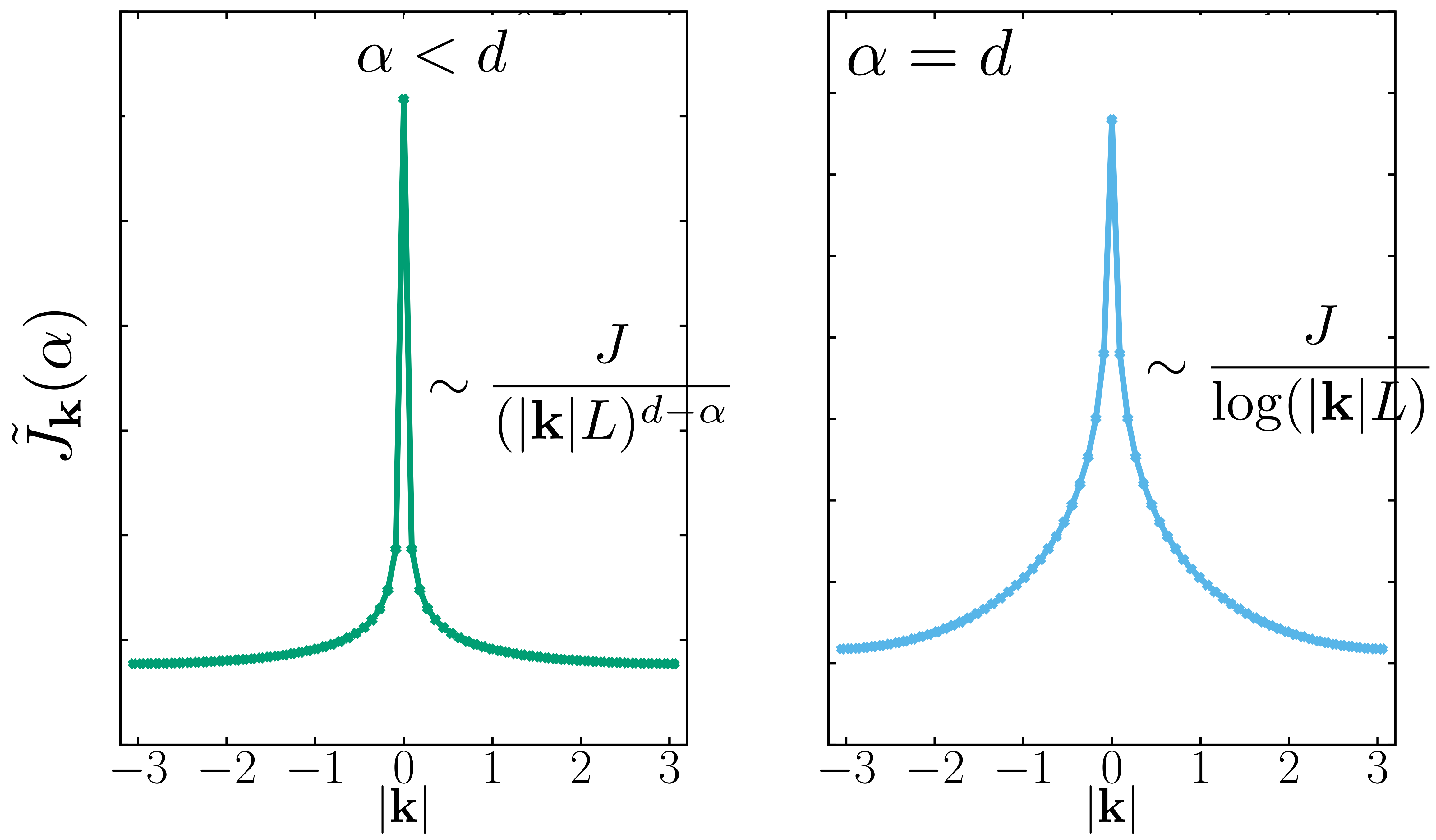
collective Hamiltonian

spin-wave Hamiltonian

—> Many-body problem!

$$\sum_{k\neq 0} \tilde{J}_k(\alpha) \left[A_{QQ} \frac{\tilde{q}_k \tilde{q}_{-k}}{2} + A_{PP} \frac{\tilde{p}_k + \tilde{p}_{-k}}{2} A_{QP} \frac{\tilde{q}_k \tilde{p}_{-k} + \tilde{p}_k \tilde{q}_{-k}}{2} \right]$$

Suppression of quasiparticle production



$$|\dot{n}_{\mathbf{k}\neq 0}(t)| = \left|\left\langle \left[n_{\mathbf{k}\neq\mathbf{0}}, \widetilde{H}(t)\right] \right\rangle\right| \sim \frac{J}{(|\mathbf{k}|L)} d - \alpha$$

long **prethermal regime:**

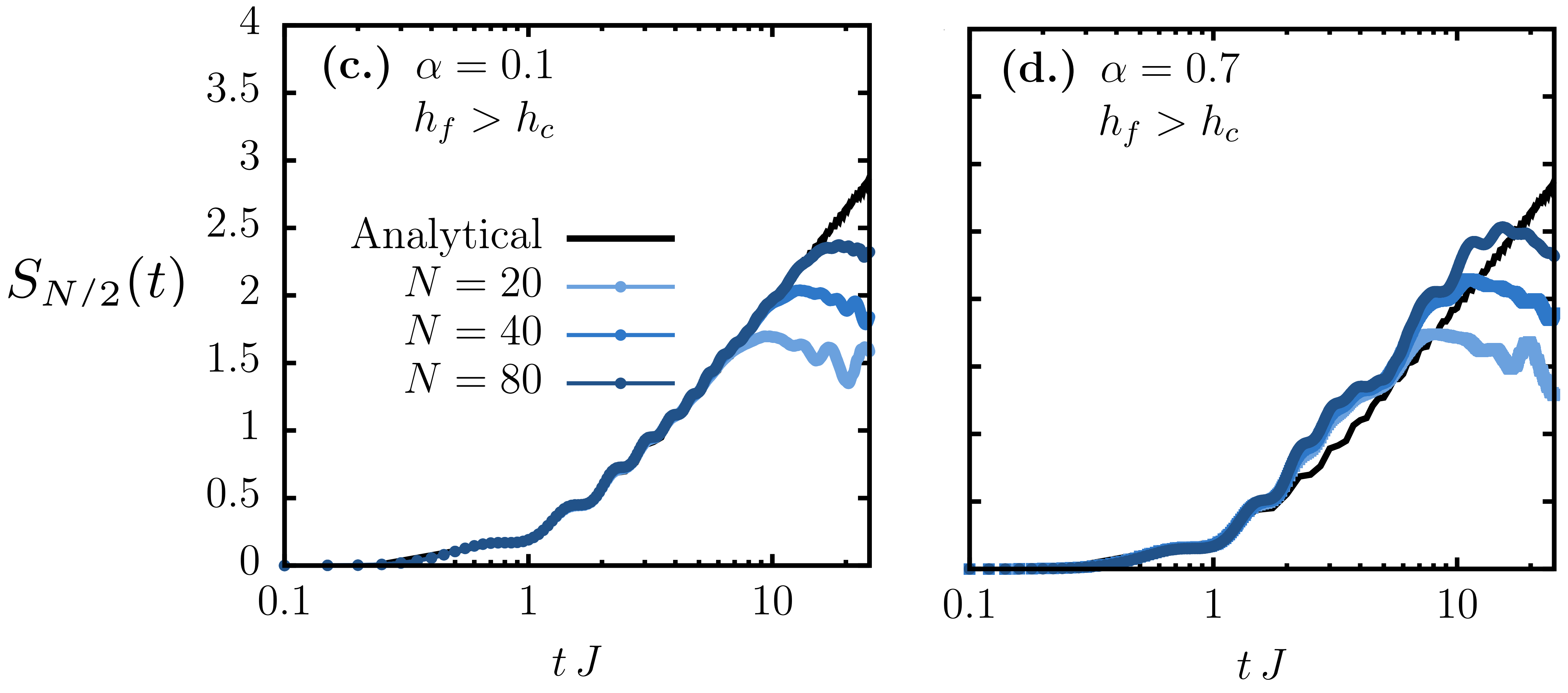
$$T_{\text{pre-th}} = N^{1-d/\alpha}$$

- Squeezing-induced entanglement dominates against quasiparticle-induced entanglement
- the system remains trapped within a small portion of the full Hilbert space

[cf. Mori - Journ. Phys. A, 2018]

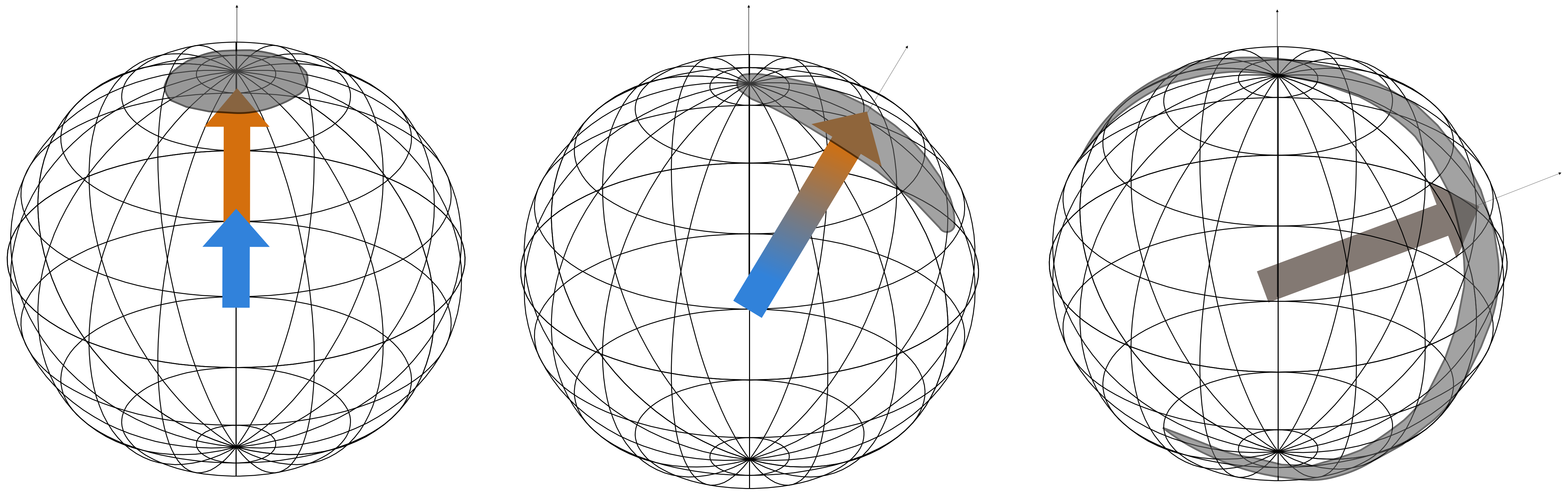
Collective squeezing vs Quasiparticle propagation

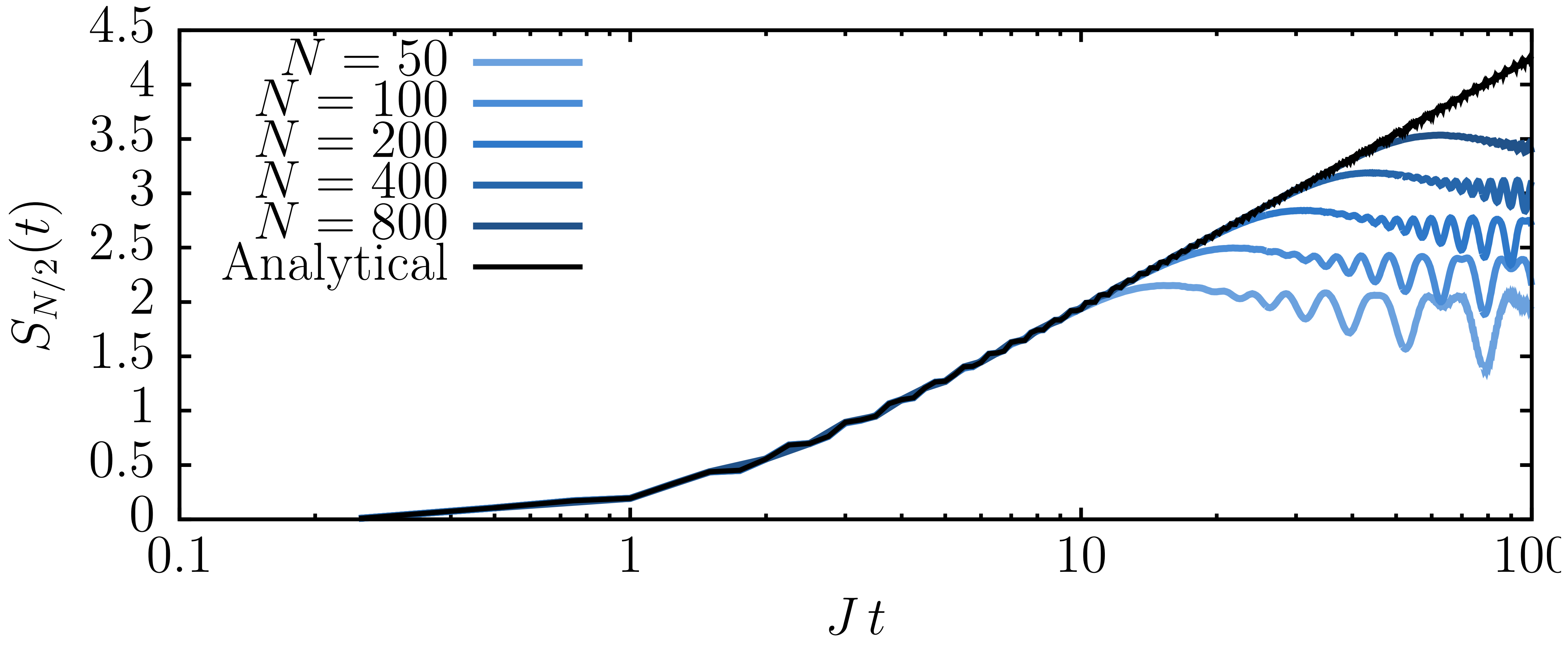
Numerical simulations by MPS-TDVP
(converged with bond dimension D=128)



- Squeezing-induced entanglement dominates against quasiparticle-induced entanglement
- Appreciable (bounded) contribution of long-wavelength quasiparticles

Conclusions





1. analytical understanding of $S_A(t)$ beyond the short-range paradigm

- collective spin squeezing gives dominant contribution
- long prethermal regime (nonergodic behavior);
- 'efficiency' of classical simulations: TDVP, CTWA etc

2. connection between $S_A(t)$ and spin squeezing

- Entanglement entropy (bound) accessible **experimentally**
- Quantum Information

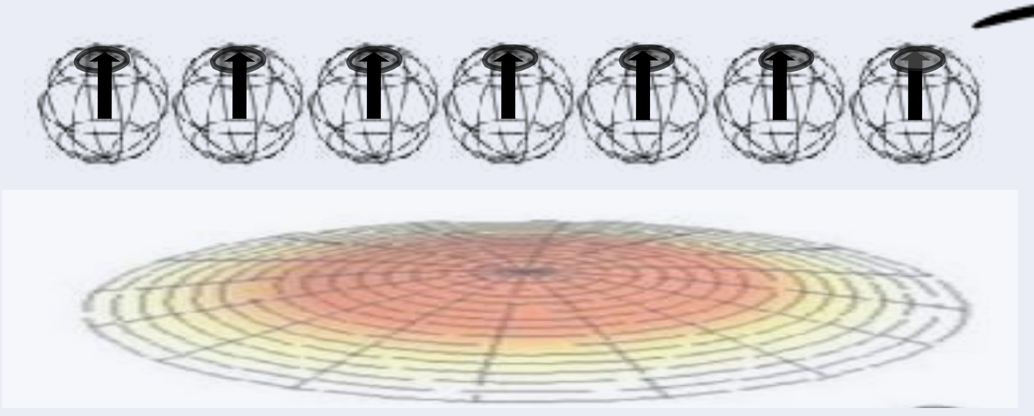
Perspectives: *entanglement dynamics in collective models*

- chaotic semi-classical models

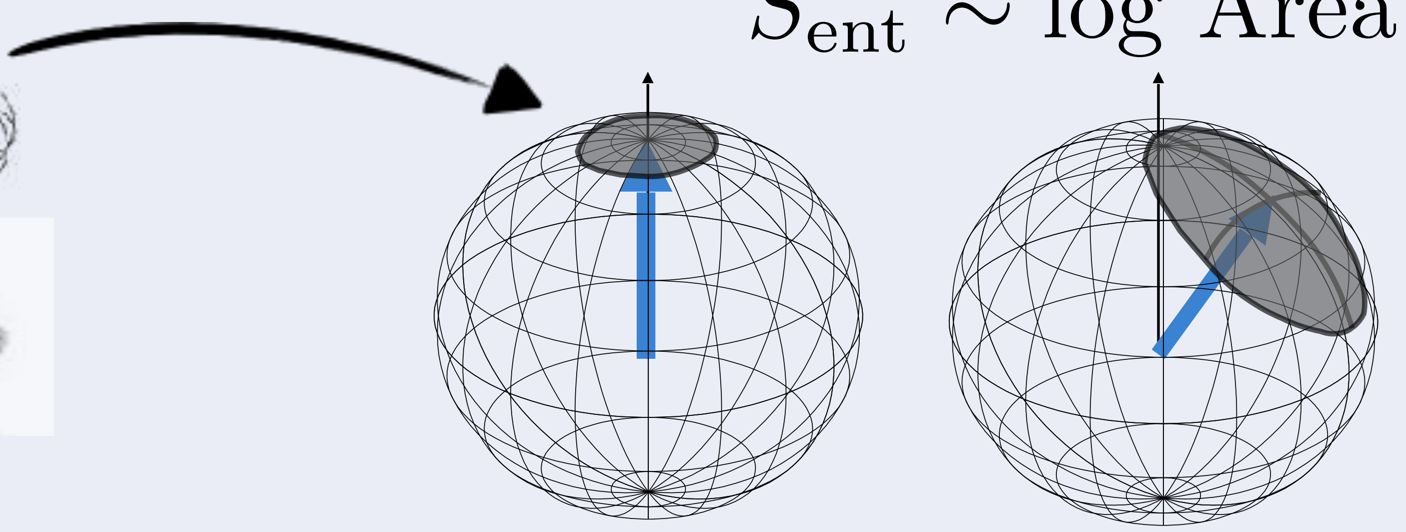
Kicked top $S_{\text{ent}} \sim \lambda_{\text{Lyap}}\, t$

- multiple collective degrees of freedom (Dicke models etc)

Dicke model



$S_{\text{ent}} \sim \log \text{Area}(t)$



Regular Phase (KAM) $S_{\text{ent}} \sim \log t$

Chaotic Phase $S_{\text{ent}} \sim (\lambda_1 + \lambda_2)\, t$

Kolmogorov-Sinai entropy

[Zurek, Paz - Physica D: Nonlinear Phenomena, 1995]

Thanks!